Homework Math 104B

1. (a) Implement Crout's method to solve a tridiagonal system of equations using the notation in Problem 27, Section 6.6. (b) Test your implementation.

2. Consider the boundary value problem:

$$\begin{aligned} -u'' + \pi^2 u &= 2\pi^2 \sin(\pi x) \quad 0 < x < 1, \\ u(0) &= u(1) = 0. \end{aligned} \tag{1}$$

   As discussed in class, we can find a numerical approximation to the solution of this problem by employing the finite difference method. Use a uniform grid with $N-1$ interior nodes to obtain, by replacing the second derivative with a second order finite difference and neglecting the (truncation) error, the linear system

$$\frac{-v_{j-1} + 2v_j - v_{j+1}}{h^2} + \pi^2 v_j = 2\pi^2 \sin(\pi x_j) \quad \text{for } j = 1, 2, \ldots, N-1. \tag{2}$$

   where h=1/N, v_j is the approximation to $u(x_j)$ for $j = 1, 2, \ldots, N-1$, and $v_0 = v_N = 0$.
   (a) Use your implementation of Crout's method to solve the tridiagonal system (2) for $N = 50$ and plot your corresponding solution. In Matlab you can use the command *plot*.
   (b) The exact solution to the boundary value problem (1) is $u(x) = sin(\pi x)$. Check this.
   (c) Compute the error of your approximation in the 2-norm for $N = 50$. Solve again (2) for $N = 100$, by how much would you expect the error to decrease? verify your answer by comparing the error for $N = 50$ and $N = 100$.

3. (Food for thought) In real applications we do not know the exact solution. What can you do to check that your method and implementation are giving you an "acceptable" approximation to the (unknown) exact solution? Hint: the method should converge to the exact solution as $h \to 0$ and the error should decrease quadratically in h.