

# Teaching Statement

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Authentic mathematics can be messy, but that messiness is what makes the work so compelling. Research mathematicians struggle with framing the problem as much as with solving it, and applications of mathematics are just as thorny since they often require balancing the accuracy of a model with ease of computation. As frustrating as these struggles can be, they also force us to be creative and they cause us to appreciate beautiful ideas. My goal in teaching is to make mathematics a more rewarding and authentic experience for my students by sharing some of the messier aspects of the subject with them. This must be done with care, as students are often unused to dealing with challenges like framing a problem or revising a hypotheses to better fit an application. I structure my classes to create opportunities for success with realistic, meaningful problems so my students will have a more positive experience with mathematics. I do this by thoughtfully introducing substantial questions, promoting student collaboration and peer discussion, and motivating students to see challenging problems through to the end. To better illustrate how this happens in my classes, I detail my approach to teaching a small inquiry-based linear algebra course and follow that with my approach to teaching more traditional large lower-division classes.

## **Math 4AI Inquiry-Based Linear Algebra**

My linear algebra course bridges the gap between lower and upper division mathematics by providing students with the opportunity to engage in genuine mathematical inquiry. Linear algebra is an ideal course for this kind of leap because this is typically when students first encounter abstraction and must wrestle with technical definitions and theorems in multiple contexts. To help my students through these challenges, we spend time on messy tasks like making observations and generalizing them, as well as learning to form and support conjectures. This focus on broader mathematics skills is supported by and in turn motivates an emphasis on technical skills like using language precisely and processing complicated definitions and theorems. I also foster habits of critical thinking by having students look to themselves to assess the validity their own ideas and those of their peers. I want my students to have the opportunity to ask and explore mathematical questions just as mathematicians do.

The course is inquiry-based, meaning that in contrast to the traditional lecture-textbook model, the course centers around a scaffolded sequence of problems that serve as the basis for class discussion, in-class group work and homework. These problems are carefully designed to guide students through the process of exploring computational examples, making observations and generalizing those observations to get to the big ideas and to inspire students to conjecture the major theorems of linear algebra. The process begins with pre-work problems assigned before each class to prepare student for the next class discussion. Students report on their progress with the pre-work online; but more importantly, they must also share their own observations and questions inspired by these problems. I then use this information to form groups for class that promote productive class discussion and cross-flow of ideas. To solidify the results of class discussion, the students work together on a large-scale collaborative writing project in the format of a wiki. They produce a comprehensive glossary of linear algebra terms that includes examples, intuitive information about terms and connections between ideas. The class wiki is also an excellent forum for students to post and support their conjectures because it makes it possible for multiple students to revise the

statement of a conjecture and any proofs or examples supporting that conjecture; moreover, these revisions can happen throughout the term, which takes much of the pressure out of the process. Students are expected to make a contribution to the class wiki after each class meeting - either new content or revisions of existing content – and this is their primary assignment outside of class.

Initially I was surprised by how challenging it was for my students to formulate conjectures, then I realized that this was happening because we were directly addressing the technical challenges inherent in linear algebra. The process of framing conjectures and alternate definitions forced students to grapple with new notation for new objects in a deeper way; and writing supporting examples for their conjectures and definitions instilled a good reasonableness-check habit in the students. Conjectures could be supported with examples OR proofs, which allowed students to make and support conjectures even if they weren't ready to formally prove them. And since the work was shared, each student could contribute at their own level. All of the conjectures came from observations students made while working computational or conceptual problems, so the theory felt like helpful patterns that emerged naturally instead of confusing ideas that had to be chewed over. In other words, the theoretical ideas were something to be discovered, instead of new roadblocks to overcome. Moreover, since any conjecture might not be true, it felt important to try to prove or disprove them. My students made tremendous progress towards understanding the purpose of mathematical precision and proof; in the process they gained ownership of the material – they were proud of their wiki in the end.

Students really enjoy this class. In an exit survey several reported that they liked collaborating with peers and that they gained from the experience. One student wrote “With this class, I could very easily explain to a student a topic but they could also help explain a different one. There was a mutual give and take that I've never really seen in any other classes.” Students also demonstrated a shift in their understanding of mathematics from something procedural to more of an exploration of ideas; one student wrote, “I've learned that the best way to learn a concept isn't to tackle an abstract problem with a formula. If you want to learn a concept, you should make yourself want to explore a common everyday problem that we can imagine and relate [to] and then scale that up into higher degrees...”

## Large Lower Division Courses

My primary goal in teaching lower division courses is to promote deeper engagement with the material so my students will appreciate that their mathematics course is a collection of beautiful ideas rather than a collection of problem solving procedures. Students often come into lower division math courses (quite reasonably) believing that the main objective in taking the course is to learn computational procedures. They tend not to have the sense that mathematics is about the ideas underneath, much less that they should be able to understand and communicate those ideas. I want my lower-division students to see some honest-to-goodness *messy* mathematics, so I have them explore examples to make generalizations and have them practice generating and communicating simple mathematical arguments. It has been my experience that having students explain their thinking can lead to powerful learning moments, so I wanted to create as many opportunities for this as possible. Given the size of a typical lower division course at UCSB (roughly 100-150 students), this goal presents challenges. My interactions with students happen in large lectures and the only “small” group interactions happen once a week with TAs. In response to these logistical hurdles, I have students working in groups on challenging discussion projects during the

section meetings with TAs to promote collaboration and give the students the chance to work on communicating basic mathematical arguments in writing, while I promote whole class discussion and student-student interactions during lecture.

Each week during section meetings the students work on discussion projects designed to get them 1) talking about challenging conceptual problems in groups with a TA there to guide them and 2) practicing writing down basic mathematical arguments and getting feedback on that work. I carefully structure these projects to have students tackle a big question in a broad way (e.g. how would you estimate the area of a funny shape), then refine their ideas by connecting them with the more standard notation and/or techniques (in this case, Riemann sums). This approach allows students to get a foothold on a complex problem before it gets technical and it helps them see the reasoning behind all the technical notation and careful wording that mathematicians use. The first discussion project from my integral calculus course has students estimate the area of Virginia from a map with scale.<sup>1</sup> They have to use an image to find upper and lower bounds for the area, then use those bounds to find an estimate that is within 5,000 square miles of the actual area (without knowing the actual area). There are lots of reasonable common-sense approaches to this problem and no math background is necessary to get started, but deeper learning comes in comparing these approaches with Riemann sums. After discussion in section and again in lecture, students work in groups to write solutions and TAs gave feedback on their ability to articulate the ideas. This final step allows students to reflect on the discussion and to practice writing simple mathematical arguments.

I also want lectures to be an opportunity for students to practice explaining their thinking, so I structure them more like discussions (students have sometimes compared my courses to an English course). The flow of lecture follows that of their discussion projects: we begin by tackling a big question in a broad way or by looking at several examples in order to make some generalizations, then I connect the students' ideas with more standard approaches. I always ask students to share their ideas at some point in the lecture – and I write those ideas down on the board so the whole class can consider them carefully. After the ideas are articulated in writing, I can ask the class whether or not they agree with their classmates; sometimes this leads to student-student discussion while they weigh the options or to a vote with clickers. If the vote is split pretty evenly, I often take the opportunity to have the students, “convince your neighbor that you are right and vote again”. This approach to lecture really empowers students to formulate their own ideas and to argue for the validity of those ideas.

My courses are ambitious and it has taken years to get them working well. This course structure relies heavily on TAs to run smoothly, and I have learned that they need support to do what I ask of them. I meet with my TAs weekly and observe them in section and give feedback at least once during the quarter. As I hone my skills for leading class discussion my student evaluations have gotten more and more positive. Students often comment that the class discussions are engaging, but more importantly, the course structure is helping them critically evaluate ideas and learn the material more deeply. In my most recent differential equations class a student wrote that the class is “very good at allowing students to arrive at their own conclusions.” Another student wrote, “I not only know how to do a problem based on some formula or method, but I also have the critical thinking skills to be able to understand why I am doing what I am doing.”

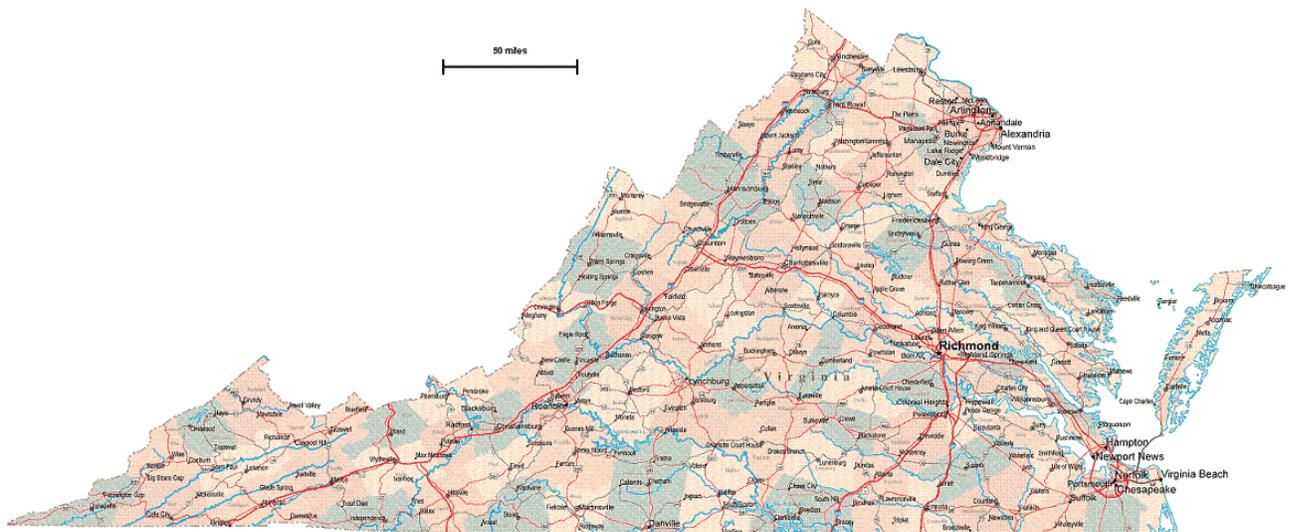
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<sup>1</sup>For those who might be interested, I have included a copy of this project at the end of this document.

# 1 Sample Calculus Project: The Area of Virginia

## 1.1 Pre-Work

Use the picture below to estimate the area of Virginia. Explain your method.



## 1.2 In Discussion

**Problem 1:** Find an *upper bound* for the area of Virginia. Explain your method and why it gives an upper bound.

**Problem 2:** Find a *lower bound* for the area of Virginia. Explain your method and why it gives a lower bound.

**Problem 3:** Estimate the area of Virginia to within 5,000 square miles. Explain how you know your estimate is good enough.