11 May 2018
M. Brown

Motivation

1. Low energy theory is exactly solved

   Strongly coupled gauge theory in 4d

2. Electromagnetic duality

   $\vec{E} \to \vec{B}$

   $\vec{B} \to -\vec{E}$

   $\mathcal{N}=4$ SYM – exact E-M duality

   $\mathcal{N}=2$: electric and magnetic particles have different spins

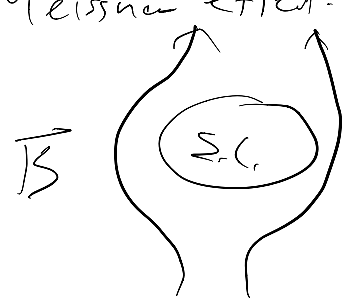
   SW has a "version" of EM duality

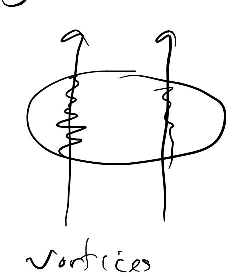
3. Confinement

   QCD: gluons + quarks

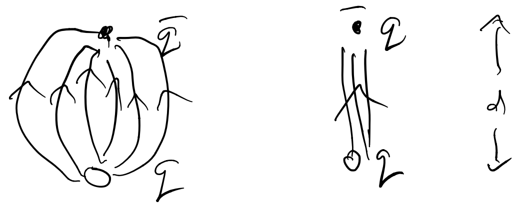
   Dual superconductor model

   Meissner effect: Screen magnetic fields





Dual: "Material" that screens electric fields
(color)



potential proportional to $\vec{d}$

Collimation of color electric flux
$\to$ confinement

Condensation

Electrons — Cooper pairs

$\langle \bar{e} e \rangle \neq 0$ condensation of electric charge

Dual: magnetic monopoles

Monopole condensation

$\downarrow$

Dual superconductivity $\to$ collimation of color electric flux

$\to$ confinement

superconductor	dual
mag flux tubes	electric flux tube
condense electric charge	

4. Math

SU(2) $\mathcal{N}=2$ topological twist

— Donaldson invariants

Low energy: abelian gauge theory

Review Formulation

SU(2) $\mathcal{N}=2$

$\mathcal{N}=2$ vector multiplet

			Spin
$i=1,2,3$	$\mathcal{N}=1$ vector	A_μ^i	1
		λ_α^i, $\chi^{\dot\alpha i}$	1/2
	$\mathcal{N}=1$ chiral	φ^i	0

Potential for the scalar field:

$$V(\varphi) = \mathrm{Tr}[\varphi^\dagger, \varphi]^2$$

Space of classical vacua (or classical moduli space)

$\Rightarrow \mathcal{M}_C$ is, up to gauge equivalence

$$V(\varphi) = 0 \Rightarrow \varphi = a\sigma_3 \quad \text{for } \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

At $a=0$, unbroken SU(2) gauge symmetry

$a \neq 0$, Higgs mechanism breaks SU(2) to U(1)

$a \leftrightarrow -a$; use $u = \mathrm{tr}\,\phi^2 = 2a^2$

$\mathcal{M}_c = \overline{\mathbb{C}}$ (the compactification of $\mathbb{C}$), with singularities.

$$\mathcal{L} = \frac{1}{4\pi} \mathrm{Im}\left(\int d^4\theta \; K(A,\bar{A}) + \int d^2\theta \, \frac{1}{2}\tau(A) W^a W_a \right)$$

vector

$F(A)$ = prepotential (essentially holomorphic)

$$K(A,\bar{A}) = \frac{\partial F}{\partial A}\bar{A}$$

$$\tau(A) = \frac{\partial^2 F}{\partial A^2}$$, the complexified gauge coupling.

$\phi = A\sigma_3$

$\mathcal{N}=1$ chiral

$\underline{a}$ scalar component of A.

$$\mathcal{L} \sim (\mathrm{Im}\,\tau)(\partial a \partial \bar{a} + F \wedge * F) + (\mathrm{Re}\,\tau) F \wedge F$$

$$\Rightarrow \quad \mathrm{Im}\,\tau \sim \frac{1}{g^2}$$

$$\tau(a) = \frac{\theta(a)}{\pi} + \frac{8\pi i}{g(a)^2}$$

The quantum moduli space $\mathcal{M}_q$ (a complex curve)

$ds^2 = (\mathrm{Im}\,\tau(a))\, da\, d\bar{a}$ (semi-simple body)

1. Holomorphy $F(A)$

2. Non-renormalization theorems

$$F(A) = \frac{1}{2}\tau_0 A^2 + \frac{i}{\pi} A^2 \log \frac{A^2}{\Lambda^2} + \frac{1}{2\pi i} A^2 \sum_{l=1}^{\infty} c_l \left(\frac{\Lambda}{A}\right)^{4l}$$

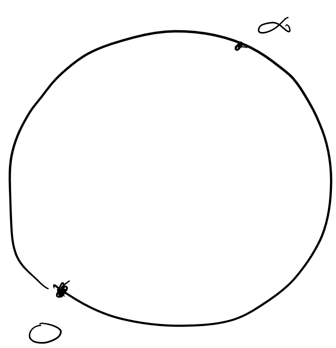
$$\tau = \frac{\partial^2 F}{\partial A^2}$$

Λ = dynamically generated scale

From the form of F, τ has monodromies.

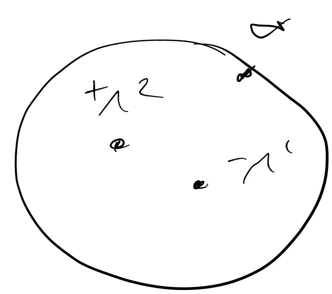
$$\tau = \text{const} + \frac{2i}{\pi} \log\left(\frac{u}{\Lambda^2}\right) + \text{single-valued}$$

Thus, $\tau \to \tau - 4$ under winding around ∞



$\mathcal{M}_c$

This is the minimal solution



$\mathcal{M}_Q$

$\mathrm{Tr}\,\varphi^2$

$u \leftrightarrow -u$

1. Get consistent monodromies

2. Get monopole condensation $\rightarrow$ confinement

3. Get $F(A)$.

Singularities in $\mathcal{M}_q$ come from particles becoming massless.

Can't be gauge bosons - must be monopoles or dyons.

$u = +\Lambda^2$, $n_m = 1$, $n_e = 0$: monopole becomes massless

$u = -\Lambda^2$ $\quad (1, -1)$

EM duality

$\mathcal{N} = 2$

EM duality, SUSY, masses of dyons

$Z = n_e a + n_m a_D$ where $a_D = \frac{\partial F}{\partial a}$

$\{ Q_i, Q_j \} \sim \delta_{ij} Z$ $\quad$ Z = SUSY central charge

BPS bound: $m^2 \geq |Z|^2$

Want: $0 = n_e a + n_m a_D = 0$

$a(u)$, $a_D(u)$

$a_D(+\Lambda^2)=0$

Want monodromies of a, a_D.

$$\tau = \frac{da_D/du}{da/du}$$

At $u=\infty$: $a \simeq \sqrt{u}$

$$a_D \simeq \frac{\sqrt{2u}}{\pi}\ln u$$

At $u=\pm\Lambda^2$: similar

$\Rightarrow a, a_D$ solved via elliptic curve

$$y^2=(x-\Lambda^2)(x+\Lambda^2)(x-u)$$

and $a_D=\oint_{\gamma_1}\lambda$, $a_D=\oint_{\gamma_2}\lambda$

λ = meromorphic 1-form.