

Based on:

Quantum Fields and strings,

A guide for mathematicians

Vol 2, Witten's Lectures 16, 17, 18

Supergravity algebras:

4D, 8 supercharges

→ "N = 2 in 4D"

→ written in terms of

~~N = 1 superfields~~

Classify reps of super algebras

∴ rep = "multiplet"

decompose into Poincaré reps

"particle types"

Superspace: Find a supermanifold which extends

$M^{3,1}$ or R^4 to a space

with even and odd coordinates.

Good ones for 2, or 4 supercharges

There is a "harmonic superspace"

for 8-supercharges.

None known for > 8 supercharges.

Gauge theory:

N = 0

N = 1

↔ N = 2

4D Gauge theory:

$$\mathcal{L} = \frac{1}{4g^2} \int \text{Tr} F \wedge * F$$

$\|F\|^2$

↑ Hodge star

A = $so(3)$ -valued gauge field.

$$F = dA + A \wedge A$$

= $so(3)$ valued curvature (2-form)

These are invariant under action of G

All physical quantities are G-invariant.

$$\tilde{\mathcal{L}} = \frac{1}{4g^2} \int \text{Tr} F \wedge * F$$

$$+ \frac{i\theta}{16\pi^2} \int \text{Tr} F \wedge F$$

N = 1 : • Curvature is part of a multiplet.

• gauge field is part of a multiplet

(A, λ, D)

↑ bosonic, quadratic in action "auxiliary field"
↑ spinor valued (fermion field)

$$\mathcal{L}_{\text{fermion}} = \frac{1}{4g^2} \int \text{Tr} F \wedge * F$$

$$+ \frac{i\theta}{16\pi^2} \int \text{Tr} F \wedge F$$

$$\left. \begin{matrix} \psi_i^- \text{ left} & \psi_i^+ \text{ right} \end{matrix} \right\} + \sum_i \left(\int \bar{\psi}_i \psi_i + \int \psi_i^* \psi_i \right)$$

$$n_f = n_f^*$$

$\mathcal{N}=1$:

$$\mathcal{L}_{\mathcal{N}=1} = \frac{1}{4g^2} \int \text{Tr} F_{\mu\nu} F^{\mu\nu} \quad (+ \text{gaugino in } D)$$

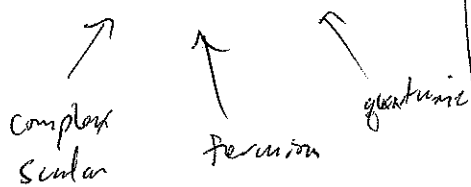
$$+ \frac{i\theta}{16\pi} \int \text{Tr} F_{\mu\nu} F^{\mu\nu} + \int \text{Tr} (\bar{\lambda} i \not{\partial} \lambda)$$

λ = adjoint valued spinor

$\mathcal{N}=1$ has "chiral superfields"

$$\Phi = (\psi, \phi, \hat{F})$$

something like
 $\psi + \theta \psi + \theta^2 \hat{F}$
 $+ \theta \theta' \hat{F}$



Term in Lagrangian:

$$\int d^4x d^4\theta \text{Tr} \Phi \bar{\Phi}$$

$$+ \int d^4x d^2\theta \text{Tr} \Phi \hat{F}$$

+ complex conjugate
 + fermion term

more generally, "superpotential polynomial" in Φ .

Remark: In $\mathcal{N}=2, \mathcal{E}=0$.

$\mathcal{N}=2$ 4D

Curvature!
 W

$$\mathcal{N}=2 \text{ vector multiplet} = \left(\begin{array}{l} \mathcal{N}=1 \text{-vector} \\ + \mathcal{N}=1 \text{-chiral} \end{array} \right)$$

$\mathcal{N}=2$ hypermultiplet

$$\bar{\Phi} \bar{\Phi} \left\{ \begin{array}{l} (\mathcal{N}=1)\text{-chiral} + (\mathcal{N}=1)\text{-chiral} \end{array} \right.$$

curvature is in a spinor valued multiplet. Key component

$$W_a \text{ (sg-valued)}$$

$$\mathcal{L}_{\mathcal{N}=2} = \frac{i}{4\pi} \int d^4x d^4\theta \bar{c}(\Phi) W_a W^a$$

$$+ \frac{1}{4\pi} \int d^4x d^4\theta K(\Phi, \bar{\Phi})$$

$$\mathcal{L} = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

varies holmorphicly with Φ .

$$\mathcal{L}_{\mathcal{N}=2} = \frac{1}{4\pi} \int_{M^4} d^4x \left(\begin{array}{l} i[\bar{c}(F^2 F^2) - c(F^2 F^2)] \\ + k_{\psi\bar{\psi}} \partial\psi\bar{\psi} \\ + (\text{Im } \tau) \bar{\lambda} i \not{\partial} \lambda \\ + K_{\phi\bar{\phi}} \bar{\psi} \not{\partial} \psi \end{array} \right)$$

$X \supseteq \mathbb{C}$

X/G

$K(\phi, \bar{\phi}) = K$ ähler potential

$K_{\phi\bar{\phi}} d\phi d\bar{\phi}$ is a Kähler metric

Sigma-Model with target $X = \text{Kähler manifold}$

$$\Phi: \mathbb{R}^4 \rightarrow X$$

$G \curvearrowright X$, Physical Theory X/G
 (solution of D-term/G) (2)