

Source: Quantum Fields & Strings:

A Course For Mathematicians

(vol. 2)
Witten's
lectures 16, 17, 18

Supersymmetry algebras:

Today is on 4D, 8 supercharges "N=2 is 4D"

These are written in terms of N=1
superfields.

Classify reps of SUSY algebras

- In reps correspond to "multiplets"
- Decompose into Poincaré reps - "particle types"

Superspace: Find a supermanifold

which extends $M^{1,3}$ or \mathbb{R}^4 to a space w/
even/odd coordinates.

These are invariant under action of G (Lie group or its algebra)

- All physical quantities are G -invariant

Now set:

$$\tilde{\mathcal{L}} = \frac{1}{4g^2} \int \text{Tr} F \wedge *F + \frac{i\theta}{16\pi^2} \int \text{Tr} F \wedge F$$

$$\boxed{N=1}$$

curvature } part of the multiplet
Gauge field

$$(A, \lambda, D)$$

Fermion field (Spinor valued)

Bosonic, quadratic in the action
"Auxiliary field"

$$\mathcal{L}_{\text{fermion}} = \frac{1}{4g^2} \int \text{Tr} F \wedge *F + \frac{i\theta}{16\pi^2} \int \text{Tr} F \wedge F$$

$$\int (\bar{\psi}_i \not{D} \psi_i^* + \psi_i \not{D} \psi_i)$$

$(n_f = n_f^*)$
 ψ_i left ψ_i^* right

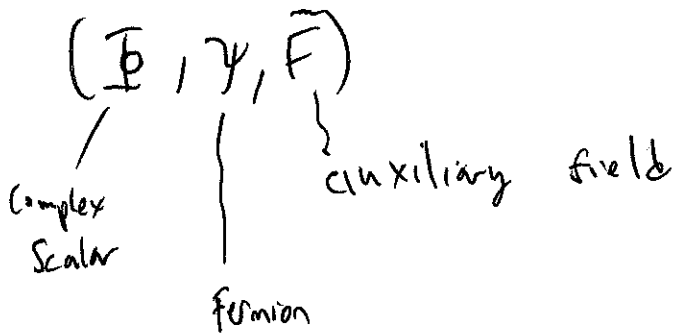
$$\underline{N=1}$$

$$\mathcal{L}_{N=1} = \frac{1}{4g^2} \int \text{Tr} F \wedge * F + \frac{i\theta}{16\pi^2} \int \text{Tr} F \wedge F + \int \text{Tr} (\bar{\lambda} i \not{D} \lambda)$$

+ something quadr. in \mathcal{D}

(λ adjoint-valued spinor)

$N=1$ also has:
chiral superfields.



Term in Lagrangian:

$$\int d^4x d^4\theta \text{Tr} \bar{\Phi} \Phi + \int d^4x d^2\theta \text{Tr} \bar{\Phi} \Phi + (\text{complex conjugate})$$

Kinetic

= Fermion term

More generally, "superpotential" polynomial in Φ

$$\boxed{N=2}$$

$N=2$ vector multiplet \leftarrow $N=1$ vector multiplet & $N=1$ chiral

$N=2$ ~~vector~~ hypermultiplet \rightsquigarrow $N=1$ chiral + $N=1$ chiral

Curvature is in a spinor-valued multiplet

Key component W_a (σ_j -valued)

$$\mathcal{L}_{N=2} = \frac{i}{4\pi} \int d^4x d^4\theta z(\Phi) W_a W^a + \frac{1}{4\pi} \int d^4x d^4\theta K(\bar{\Phi}, \Phi)$$

$$z = \frac{\Theta}{2\pi} + \frac{4\pi i}{g^2} \quad \sim \text{varies holomorphically w/ } \Phi$$

$$\mathcal{L}_{N=2} = \frac{1}{4\pi} \int_{M^4} d^4x \left(i [\bar{z} (F^+ \wedge F^+) - z (F^- \wedge F^-)] + \right. \\ \left. K_{\phi\bar{\phi}} \partial\phi \partial\bar{\phi} + (Im z) \bar{\lambda}_i \partial_A \lambda^i + \right. \\ \left. * F^{\pm} = \pm F^{\pm} \right. \\ \left. K_{\psi\bar{\psi}} \bar{\Psi}_i \partial_A \Psi^i \right)$$

$K(\phi, \bar{\phi}) =$ Kähler potential.

~~then~~ $K_{\phi\bar{\phi}} d\phi d\bar{\phi}$ is a Kähler metric

Sigma model; w/ target X a Kähler manifold.

$$\Phi: \mathbb{R}^4 \rightarrow X$$

G acts on X

Physical theory only sees G -invariant things.

$$X/G \supseteq \text{Solutions of D-terms} / G$$

$$\begin{array}{c} X \supset G \\ \vdots \\ X // G \end{array} \quad \text{symplectic red.}$$