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Q. system = f-d Hilbert space $\mathcal{H}$

Q. state = $|\psi\rangle \in \mathcal{H}$ up to a factor

Q. system on space Y $(\mathbb{R}^n, D^n)$

Set of "atoms" A

each "atom" is $\mathcal{H}_j$

$\vec{r} : A \to Y$

Nice quantum state = ?

IQHE

FQHE ...

Fermions

$\mathcal{H} = \mathcal{H}^{(0)} \oplus \mathcal{H}^{(1)}$

$\mathrm{SWAP}_{gr} : \mathcal{A} \otimes \mathcal{B} \to \mathcal{B} \otimes \mathcal{A}$

$\quad \parallel$

$\sum_{\alpha,\beta} (-1)^{\alpha\beta}\, \mathrm{SWAP}(\mathcal{A}^{(\alpha)}, \mathcal{B}^{(\beta)})$

Free fermions

$\mathcal{L} = \mathbb{C}^M \qquad FS(\mathcal{L}) = \bigoplus_{k=0}^{M} \Lambda^k \mathcal{L}$

$|0\rangle,\ a_1^\dagger, \ldots, a_M^\dagger \qquad (a_1^\dagger)^{n_1} \cdots (a_M^\dagger)^{n_M} |0\rangle$

$$a_j a_k^\dagger + a_k^\dagger a_j = \delta_{jk}$$

X is an operator on $\mathcal{L}$

$X^\dagger = X$, $X^2 = 1$

$\mathcal{M} \subseteq \mathcal{L}$ eigenspace for e.v. $= -1$

$f_1^\dagger, \dots, f_m^\dagger$ basis of $\mathcal{M}$

$$|\psi(X)\rangle = f_1^\dagger \cdots f_m^\dagger |0\rangle$$

H nondegenerate, $H^\dagger = H$

$$\hat{H} = \sum_{j,k} h_{j,k} \hat{a}_j^\dagger \hat{a}_k$$

$$X = \operatorname{sgn}(H)$$

$$\xi_1, \dots, \xi_M$$

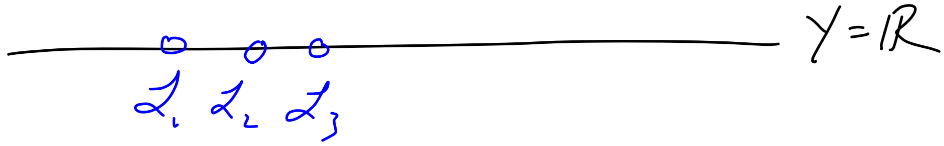
$$\varepsilon_1, \dots, \varepsilon_M$$

Space of nondegenerate H

$$\downarrow$$

space of f-f states

$$\mathcal{F} = \bigcup_{m=0}^{M} U(M) / U(m) \times U(M-m)$$

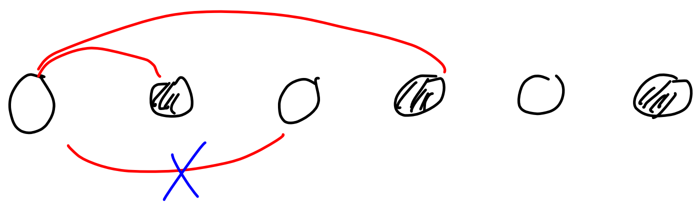


1) H is ℓ-local

$$H_{jk} = 0 \text{ if } |\vec{r}_j - \vec{r}_k| > \ell$$

2) H is gapped

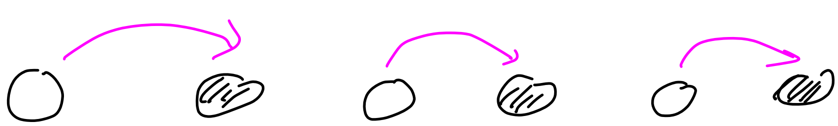
$$\operatorname{Spec} H \subseteq [-\Lambda, -\Delta] \cup [\Delta, \Lambda]$$



$$H = \left(\begin{array}{c|c} 0 & A \\ \hline A^{+} & 0 \end{array}\right)$$

$$\operatorname{Spec} A^{+}A \subseteq [\Delta^2, \Lambda^2]$$

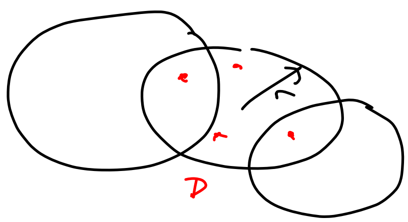
$$A|j\rangle = |j+1\rangle$$



$$\Delta^2 \leq H^2 \leq \Lambda^2$$

$$1 - \delta \leq H^2 \leq 1 + \delta$$

$$\|\underbrace{H^2 - 1}_{B}\| \leq \delta$$



$$\Pi_D (H^2 - 1) \Pi_D \leq \delta$$

Lemma

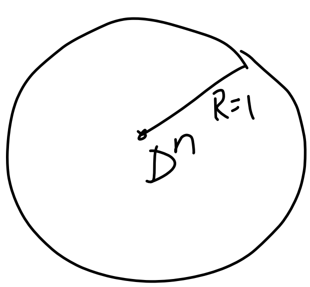
Let B be ℓ-local

$\| \Pi_D B \Pi_D \| \leq \delta$ for all disks D of radius r

s.t. $D \cap \mathrm{Supp}\, B \neq 0$

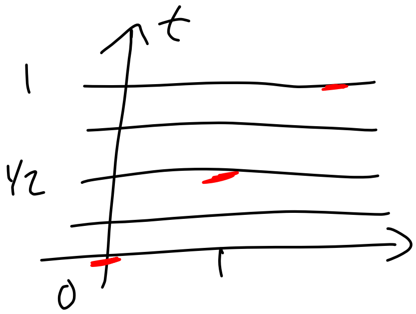
Then $\|B\| \leq \delta \cdot \left(1 + O\left(\frac{\ell^2}{r^2}\right)\right)$

$f(H)$



$$\mathcal{F}_n^{\binom{\text{free}}{U(1)}} \sim \begin{cases} BU \times \mathbb{Z} & \text{if } n \text{ is even} \\ U & \text{if } n \text{ is odd} \end{cases}$$

$$\mathcal{F}_0^{\binom{\text{free}}{U(1)}} = \bigcup_m U(M) / U(m) \times U(M-m)$$



$$\mathcal{M}_j^{(0)} \subseteq \mathcal{L}_j$$

$$\begin{array}{ccc} \mathcal{M}_1^{(0)} & \subseteq & \mathcal{L}_1 \\ \cap & & \cap \\ \mathcal{M}_2^{(0)} & \subseteq & \mathcal{L}_2 \\ \vdots & & \vdots \end{array}$$

$$m - m_0 \longrightarrow \mathbb{Z}$$

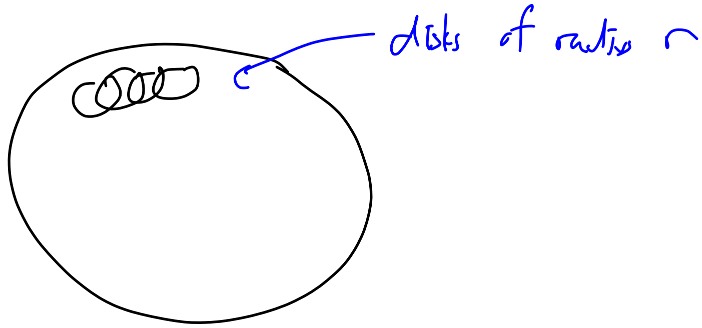
$$\lim \bigcup_m U(M)/_{U(m)\times U(M-m)} \longrightarrow BU$$

$$\begin{pmatrix} H & 0 \\ 0 & -H \end{pmatrix} \sim \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

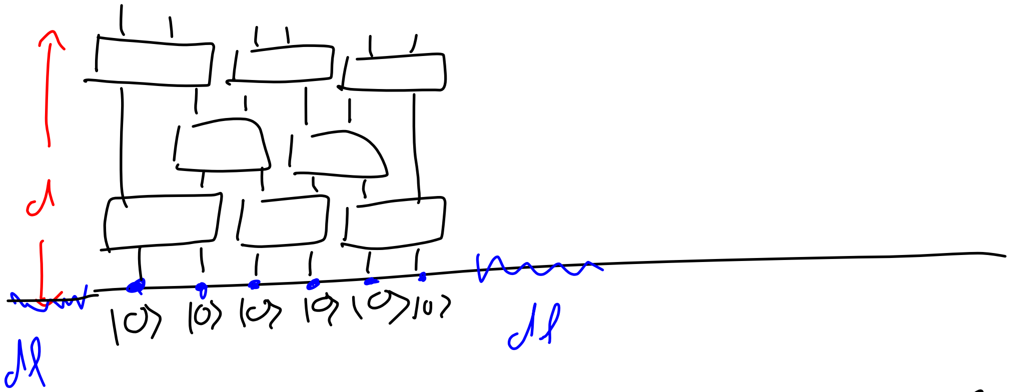
$$B(t) = \underbrace{\begin{pmatrix} H & 0 \\ 0 & -H \end{pmatrix} \cos \frac{\pi t}{2}}_{X} + \underbrace{\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \sin \frac{\pi t}{2}}_{Y}$$

$$(X+Y)^2 = X^2 + Y^2 + (XY + YX)$$

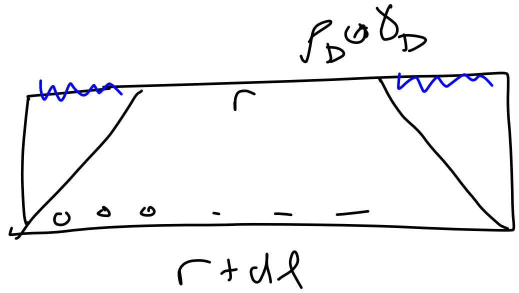
<u>SRF state</u>

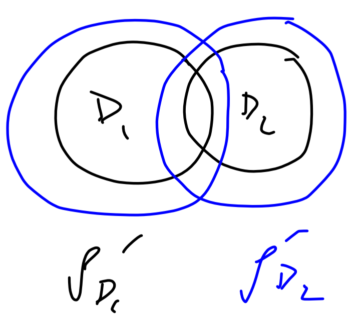


$\rho_D \otimes \gamma_D$ $\square$



ρ_D is (ℓ, d, ϵ)-constructible if $\exists\ \gamma_D$ and circuit C_D





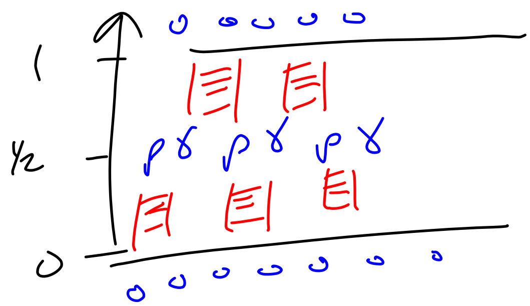
$\mathcal{B}_n$ – Bose (spin) SRE states in D^n

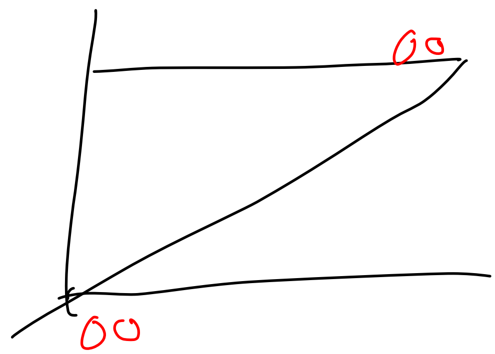
$\mathcal{F}_n$ – Fermi systems

$\mathcal{B}$ $\quad \mathcal{B}_n \sim \Omega(\mathcal{B}_{n+1})$

$\alpha : B_n \to \Omega(B_{n+1})$ $\qquad$ $\beta : \Omega(B_{n+1}) \to B_n$

ρ, γ $\qquad$ $n = 0$





$0 \; 0 \; \rho \, \gamma \, \rho \, \gamma \, \rho \; 0 \; 0$

$B_0 = \mathbb{CP}^\infty, \qquad B_1 = K(\mathbb{Z}, 3)$