25 October 2013
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# Matter from geometry without resolution

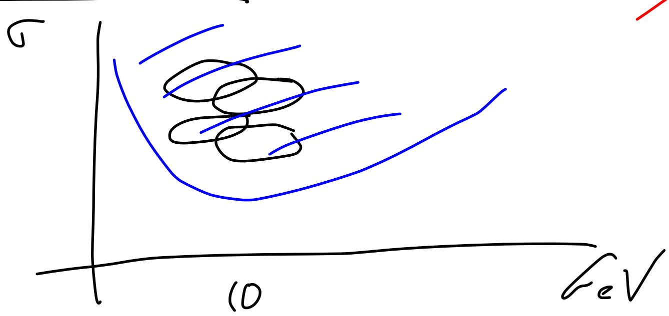
w/ A. Grassi, J. Shaneson: arXiv:1306.1832

see also: Katz-Vafa '96
Grassi-Morrison '00, '11
Morrison-Taylor '11
Simons talk
Github code

LHC Higgs boson + nothing

Dark matter direct detection

> matter




Top-down Q: how are stringy gauge theories ⊆ larger set of gauge theories

What matter representations?
limited in the string landscape

Matter from geometry

* M-theory on $G_2$ manifolds
* F-theory on $E \hookrightarrow Y_d \twoheadrightarrow B_{d-1}$

## Why F-theory?

(1) It exists

(2) landscape

(3) Geometry / gauge theory / pure math

$$\begin{tikzcd} & M \arrow[d, "F = M|_{A\to 0}"] & \\ & F & \\ \text{Het} \arrow[ur] & & \text{IIB} \arrow[ul, "g_s \ll 1"'] \end{tikzcd}$$

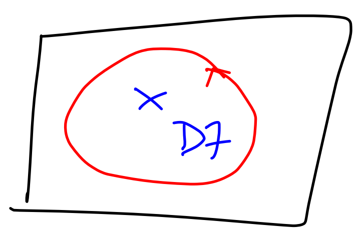
$\boxed{\text{without resolution}}$

physics argument $\longrightarrow$ $\exists$ math structures

$\mathbb{Z}^N$ ADE representations

## View from type IIB

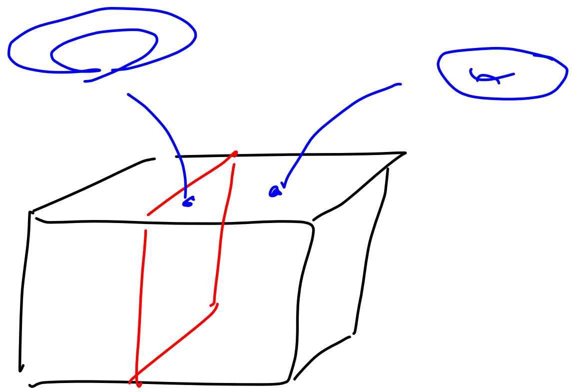
$$\tau = C_0 + \frac{i}{g_s}$$




$\tau \to \tau + 1$

$$SL(2,\mathbb{Z}): \quad \tau \to \frac{a\tau + b}{c\tau + d}$$

(2) E-fib



<u>Weierstrass form</u> $y^2 = x^3 + fxw^4 + gw^6$

$(x, y, w) \in \mathbb{P}_{(2,3,1)}$

$$f \in \Gamma(K_B^{-4}), \quad g \in \Gamma(K_B^{-6})$$

<u>$w=0$</u> $y^2 = V_3(x)$

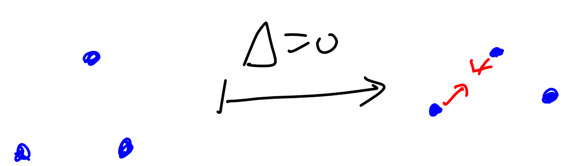
<u>Ex</u> $B = \mathbb{P}^1$, $f = f_8$, $g = g_{12}$

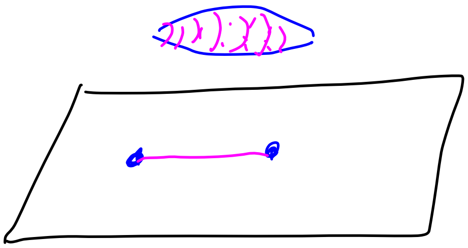
(3) <u>Singular fibers</u>

$F(x,y) = 0$ $\quad F = \partial_x F = \partial_y F = 0 \Rightarrow$ Singular fiber

occurs when $\Delta = 4f^3 + 27g^2$, $\Delta = 0$

Alternatively, $y^2 = V_3(x)$ is singular if $V_3(x)$ has a multiple root



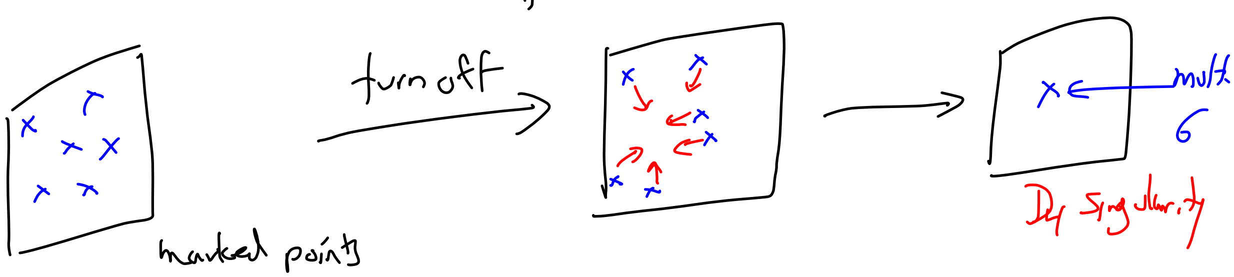


④ Kodaira $\Delta = z^N \tilde{\Delta}$

| ord(f) | ord(g) | $N \equiv$ ord($\Delta$) | fib | Sing |
|---|---|---|---|---|
| 0 | 0 | $n$ | $I_n$ | $A_{n-1}$ |
| 2 | 3 | $n+6$ | $I_n^*$ | $D_{n+4}$ |
| $\geq 3$ | 4 | 8 | $IV^*$ | $E_6$ |
| 3 | $\geq 5$ | 9 | $III^*$ | $E_7$ |
| $\geq 4$ | 5 | 10 | $II^*$ | $E_8$ |

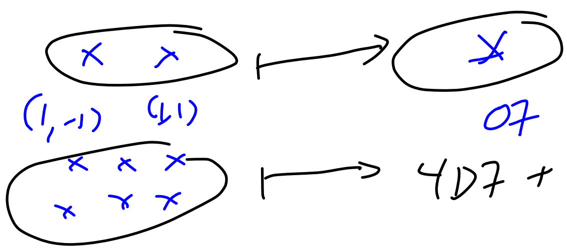
NB: simply laced only: no $B_n, C_n, F_4, G_2$

e.g. $B = \mathbb{P}^1$ $\quad \Delta \in \Gamma(\mathcal{O}_{\mathbb{P}^1}(24))$



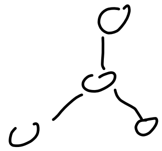

Why $D_4$?

① IIB $g_s \ll 1$, $\quad 24 \to 16\, D7 + 4\, O_7$



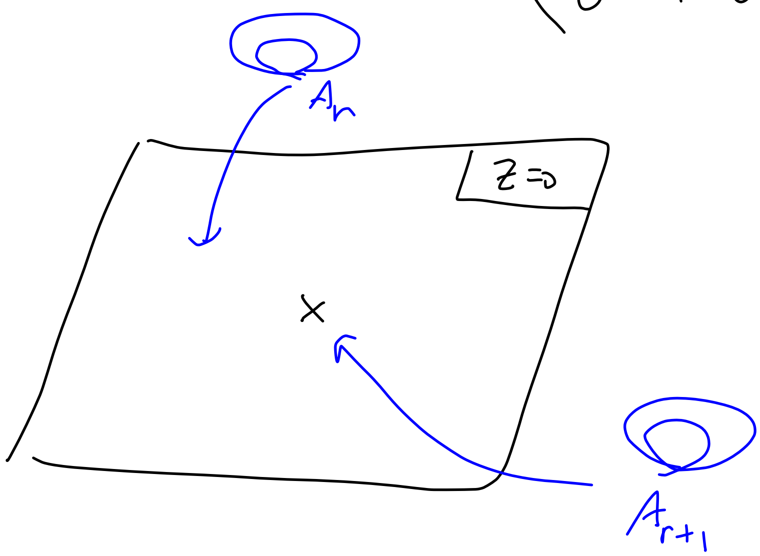

$\longmapsto 4D7 + O7 \longrightarrow SO(8)$

## M-theory blowups

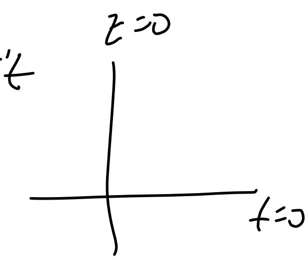


resolution curves

$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

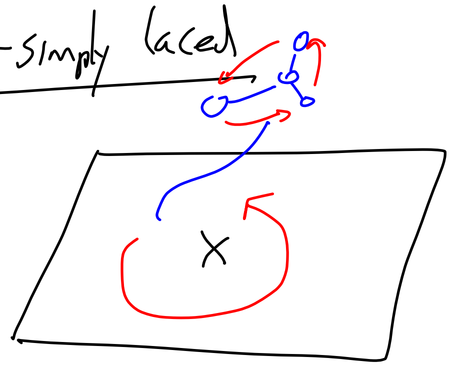



$\Delta \sim z^{r+1} t$

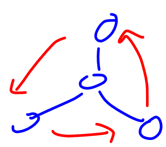



$$\mathrm{Adj}(A_{r+1}) \to \mathrm{Adj}(A_r) + \square_{A_r} + \bar{\square}_{A_r} + 1$$

## Non-simply laced



$\mathbb{Z}_3$-automorphism

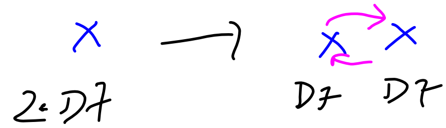


$\to$ identify outer nodes

$D_4 \to G_2$

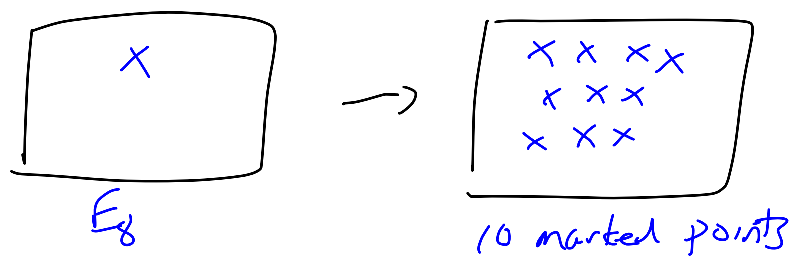
## Objections

IIB




Heterotic dual: $E_8$ vector bosons $\longmapsto$ 240 massive bosons

but these vector bosons $\xmapsto{\text{het}/F}$ cx structure moduli (not Kähler)




## Physics predictions

$\exists$ 240 W-bosons from the deformation

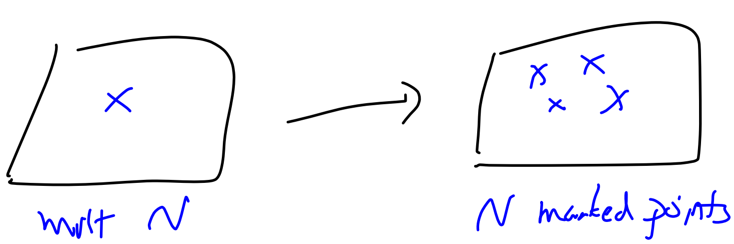
$\Rightarrow$ roots in $\mathbb{Z}^N$ $\quad (N \neq rk(G))$

$N$ is NEVER the rank.

for $E_8$, $N_{E_8} = 10$

## general case

$\Delta \sim z^N$



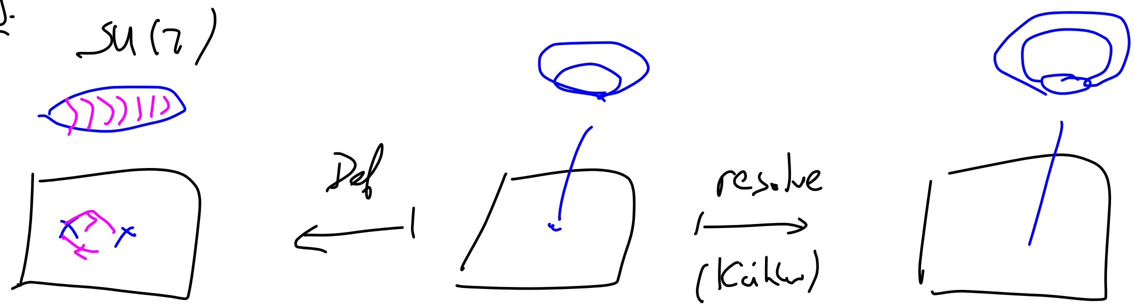

① # marked points = # 7-branes

② marked points $\longleftrightarrow$ 2-cycles $\pi_2$

③ $G$ is Higgsed

④ There are not resolution cycles

e.g.

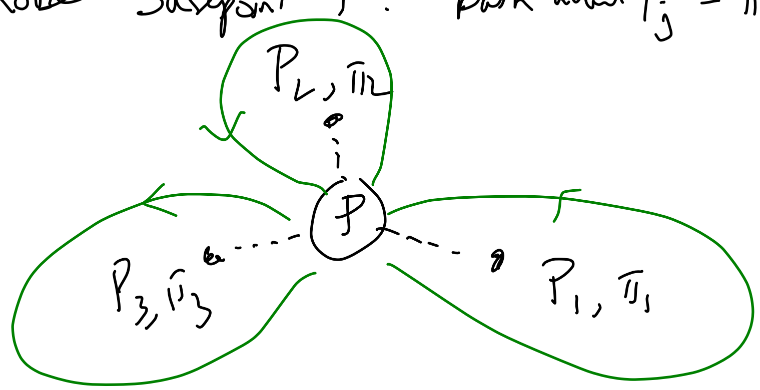



## Steps

① deform s.t. $Z \sim \Delta^N \longmapsto \Delta \sim \prod_{i=1}^{N} (z - p_i)$,

$p_i \neq p_j \quad \forall i \neq j$

② choose basepoint $P$. path around $p_j = \pi_j$




—— = example path $Z = \{\pi_1, \pi_2, \pi_3\}$

(NB: the order matters)

another order $Z = \{\pi_1, \pi_3, \pi_2\}$

path $\qquad Z = \{\Pi_i\}$ ordered set

Def
$$\Pi_A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \Pi_B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \Pi_C = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

rep 2-mfld BJ $\qquad J = (J_1, \dots, J_N) \in \mathbb{Z}^N$

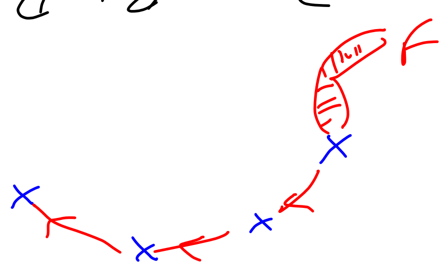
asymptotic charge $\qquad a(J) = \sum_i J_i \Pi_i$

$$\alpha_i = (1, -1, 0, 0, 0) \qquad a(\alpha_i) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$F = (1, 0, 0, 0, 0) \in \Gamma$$

$$( \,,\, ) : \mathbb{Z}^N \times \mathbb{Z}^N \to \mathbb{Z} \qquad (\alpha_i, \alpha_j) = -A_{ij}$$

e.g.



Zwiebach-DeWolfe $\qquad$ "canonical basis"

$$Z_{D_4} = \{\underbrace{\Pi_A, \dots, \Pi_A}_{4}, \Pi_B, \Pi_C\}$$

Freudenthal's formula

126 of $SO(10)$

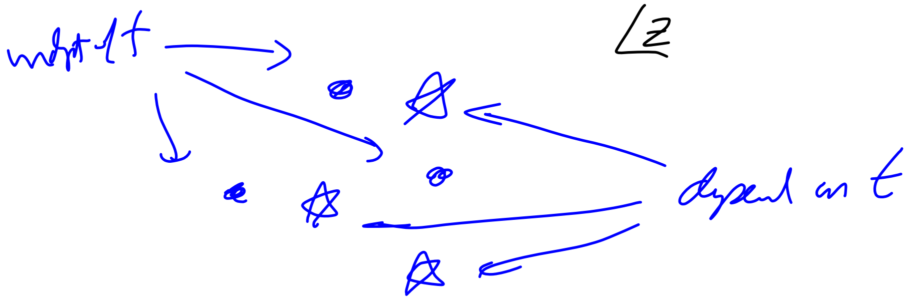
43,758 of $E_6$

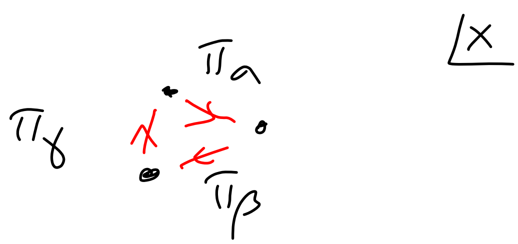
$F : \mathbb{Z}^N \to \mathbb{Z}^r \qquad$ junc $\to$ Dynkin basis

$D^4 \to G_2$

$$y^2 = x^3 - 3c^2z^3x + 2c^3z^3 + atz^3 + \epsilon t$$

$$\Delta = \left[(at + 4c^3)z^3 + \epsilon t\right](az^3 + \epsilon)$$

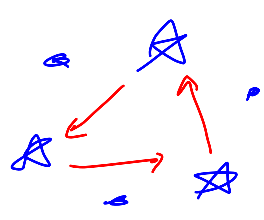



$$Z_{t=0} = \{\Pi_\alpha, \Pi_\beta, \Pi_\gamma, \Pi_\alpha, \Pi_\beta, \Pi_\gamma\}$$

Leads to $D_4$ algebra.

$\epsilon \to e^{pi} t$ : track 1-cycles

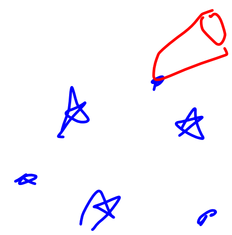


Vanishing cycle at $p = 2\pi$

$$Z_{2\pi} = \{\beta\,\gamma\,\alpha\,\beta\,\gamma\,\alpha\}$$

$$Z_{4\pi} = \{\gamma\,\alpha\,\beta\,\gamma\,\alpha\,\beta\}$$

Simple one



$J_\rho$

$(J_\rho, J_\rho) = -1$

$$a(J_0) = \Pi_\alpha$$

$$a(J_{2\pi}) = \Pi_\beta$$

$$a(J_{4\pi}) = \Pi_\gamma$$

$J_0 \in \delta_V$, $J_{2\pi} \in \delta_S$, $J_{4\pi} \in \delta_C$

permutes outer nodes, $D_4 \to S_2$