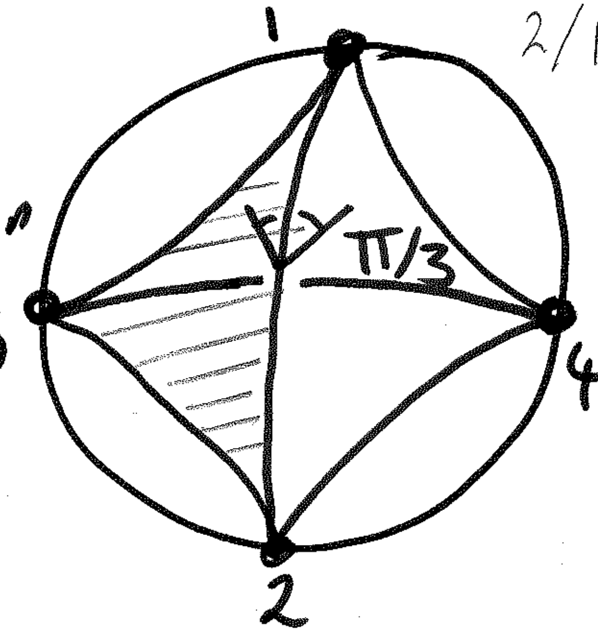
Tesselate $\mathbb{H}^3$ reflect across faces of T

T = Regular ideal tetrahedron

dihedral angles $= \frac{\pi}{3}$

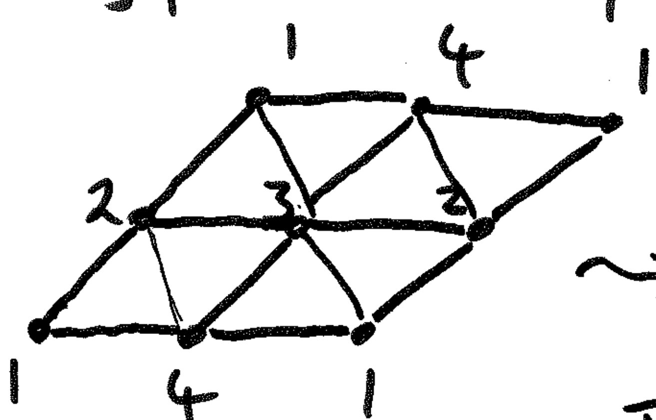


- 6 tet. round each edge
- G = all symmetries of tiling

$\supset G'$ = coxeter group $\tau_1, \dots, \tau_4$ $\quad \tau_i^2 = 1 = (\tau_i \tau_j)^3$

$\supset G''$ $\quad \mathbb{H}^3 / G'' = S^3 -$  $= M_8$

P = hyperbolic plane containing (face 4)

$=$ 

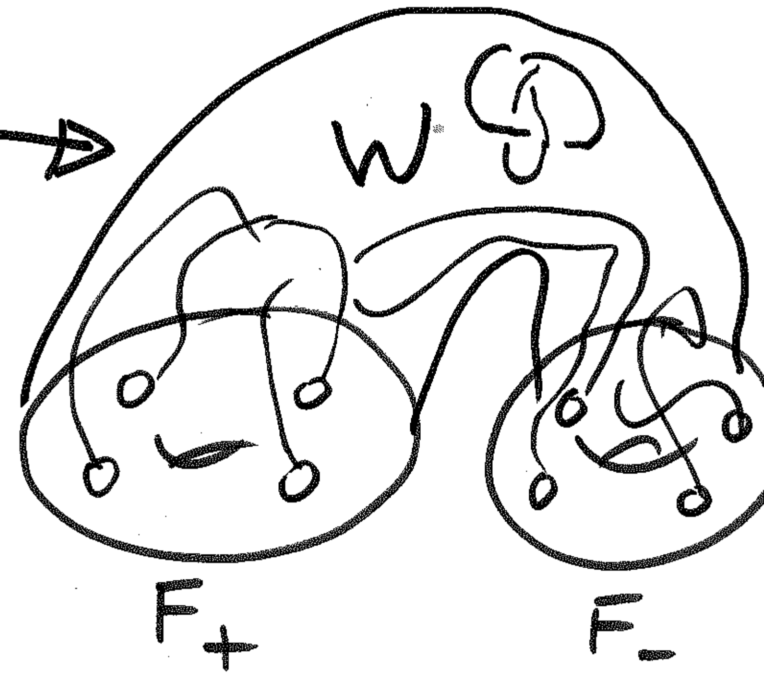
$\leadsto$  $= F \subset N$

Torus + 4 punctures

Darren Long

$N \xrightarrow{\text{finite cover}} M_8$

Cut N along F →



- W = Finite volume hyp 3-mfd
- ∂W totally geodesic

annular cusps + (rank 2 cusps)

- triangulated by <u>regular</u> ideal tetrahedra
- Cut open cover of figure 8 knot complement

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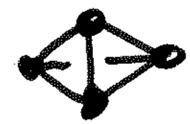
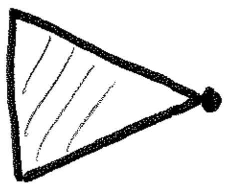
Pachner Moves

n-simplex := convex hull of $(n+1)$ points in general position in $\mathbb{R}^N$



$$\Delta^{n=a+b+1} = \Delta^a * \Delta^b = A * B$$

$A = \Delta^1$



$B = \Delta^0$

$$\partial(A*B) = \underbrace{(\partial A)*B}_{(a+1)} \cup \underbrace{(A*\partial B)}_{(b+1)}$$

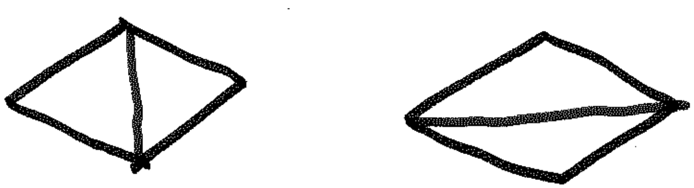
$n-1$ simplices

$$(a+1) \xrightarrow[\text{move}]{\text{Pachner}} (b+1)$$

Replace $(\partial A)*B$ by $A*(\partial B)$

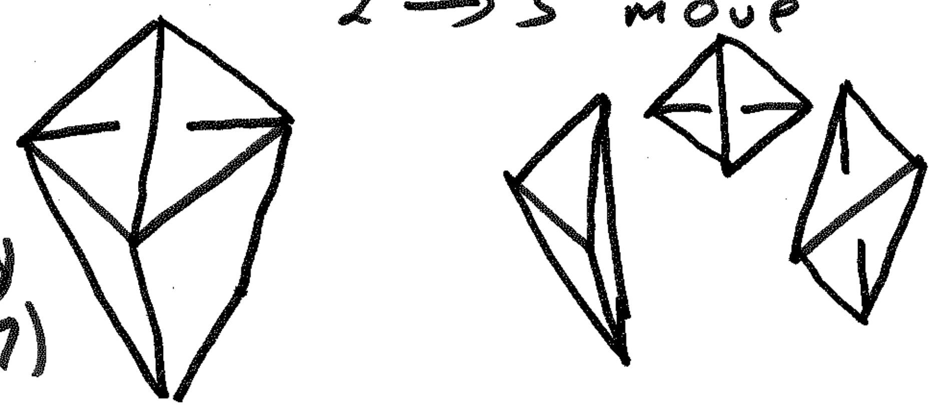
$\Leftrightarrow$ glue on simplex of 1 higher dimension along $(\partial A)*B$

$2 \to 2$ move



edge flip

$2 \to 3$ move



Newman (1926)
Pachner (1987)

Suffice for combinatorial Δ^n manifolds

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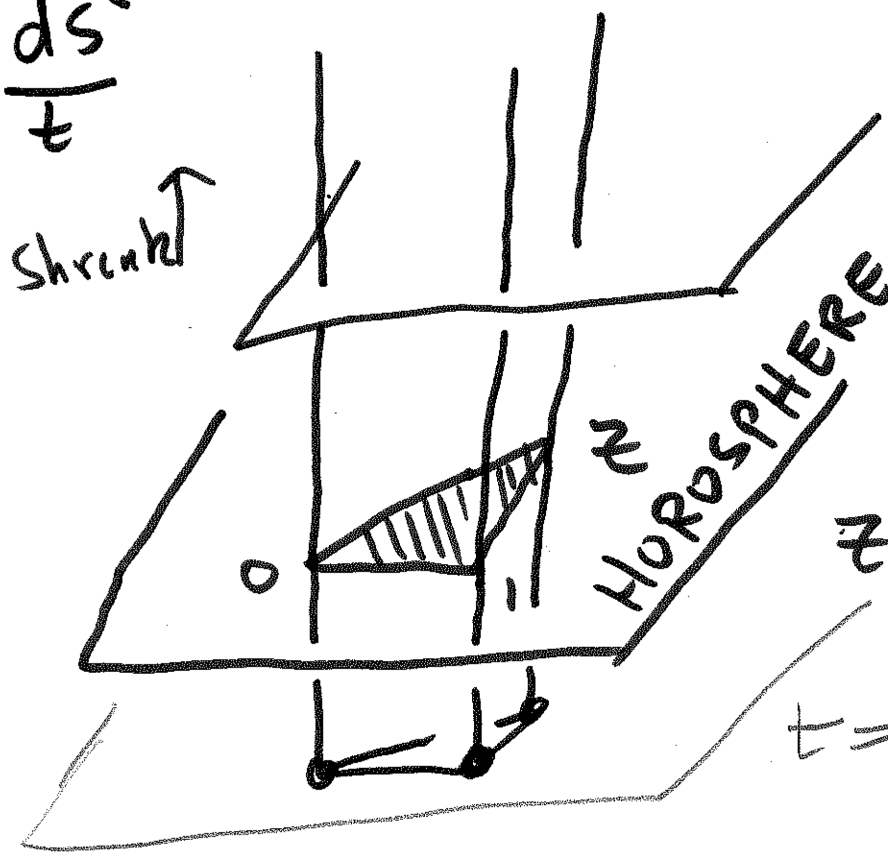
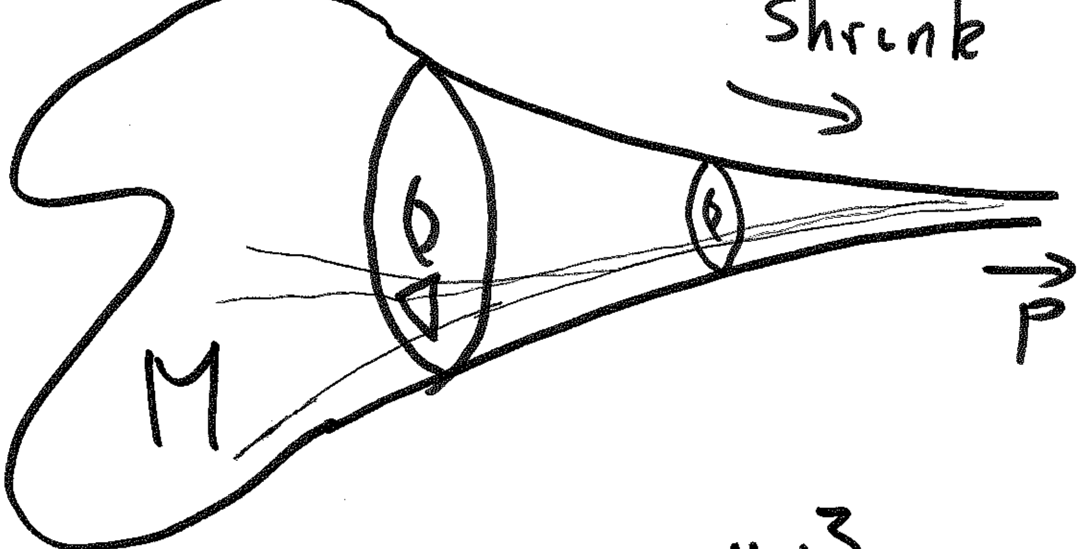
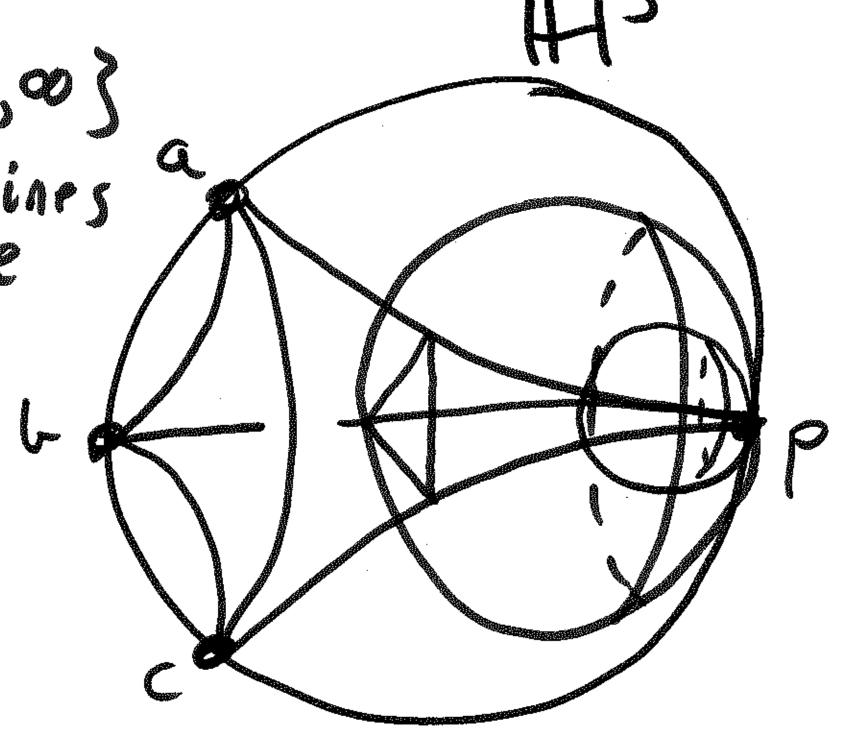
$$\frac{ds^2}{t}$$



$z \in \mathbb{C} - \{0,1,\infty\}$ ← determines shape ideal tet.





Isometries fix p

$$\longleftrightarrow \begin{pmatrix} \lambda & a \\ 0 & \lambda^{-1} \end{pmatrix} \quad z \longmapsto \lambda^2 z + \lambda^{-1} a$$

$\longleftrightarrow$ EUCLIDEAN SIMILARITIES

Complete cusp $\longleftrightarrow$ SIMILATIES ARE ISOMETRIES

$\Longleftrightarrow \lambda \equiv 1$

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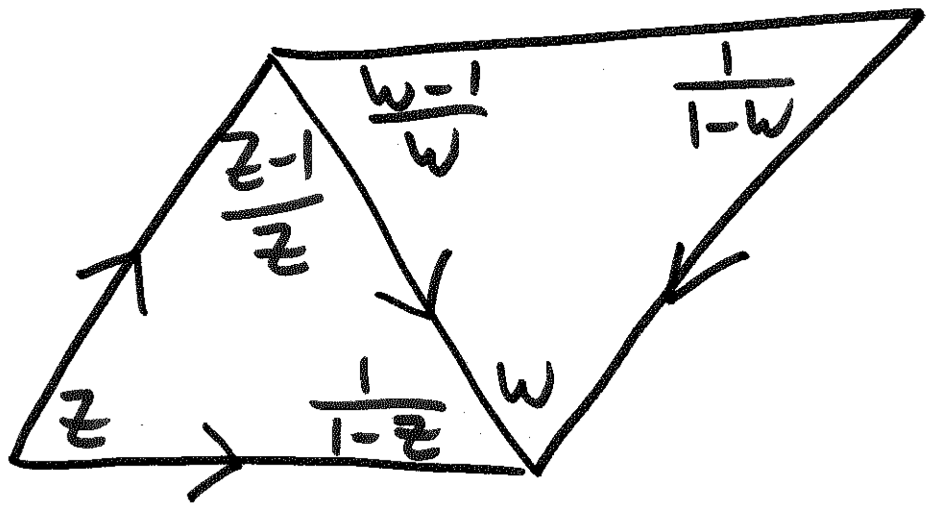
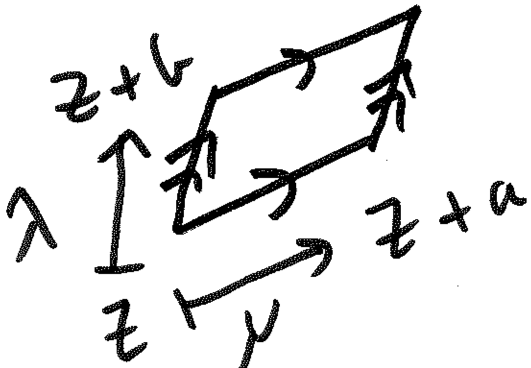
Completeness Condition



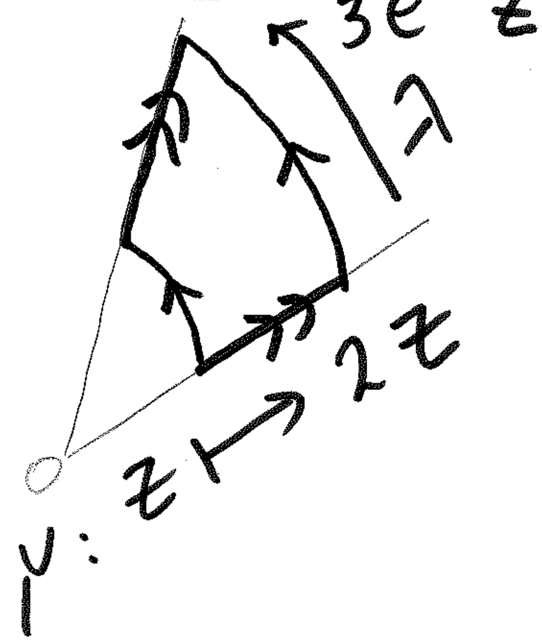
Linear part of holonomy is trivial

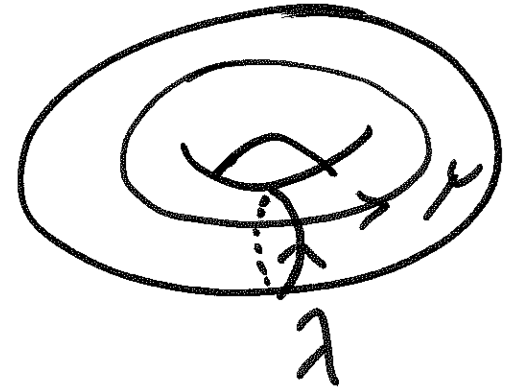
$$\arg\left(z^{-1}\cdot\left(\frac{1}{1-z}\right)^{-1}\cdot w^{-1}\right)=\pi$$

Similarity Structures

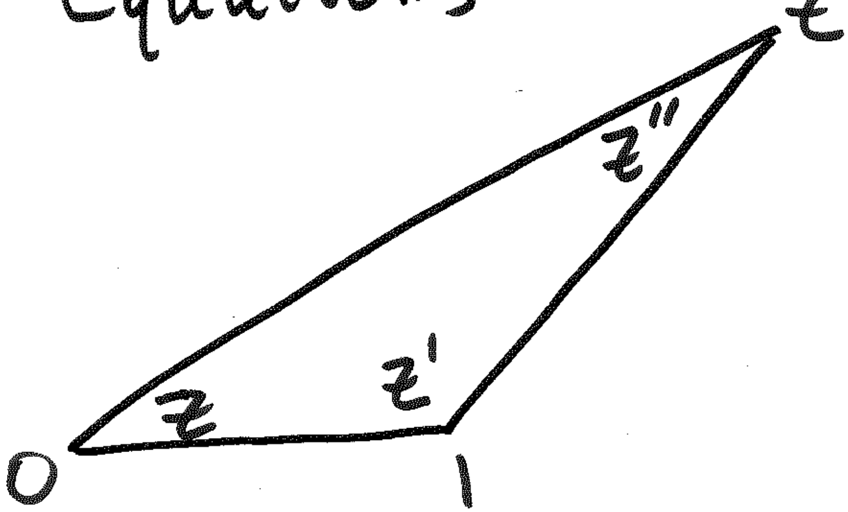


Euclidean





Equations



$$z' = \frac{1}{1-z} \qquad z'' = \frac{z-1}{z}$$

$$z z' z'' = -1$$ phase space

$$(1-z)z' = 1$$ Lagrangian

Similarity str $\Longleftrightarrow$ $\prod_{\text{vertex}} z_i^{(\ )} = 1 \quad \left(\sum \log = 2\pi i\right)$

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Fock-Gonacherov Coords

Higgs bundles
Hitchin compt

Real projective structure

$$\mathcal{T}_{\mathbb{RP}}(F) \cong \mathbb{R}^{8|\chi(F)|}$$

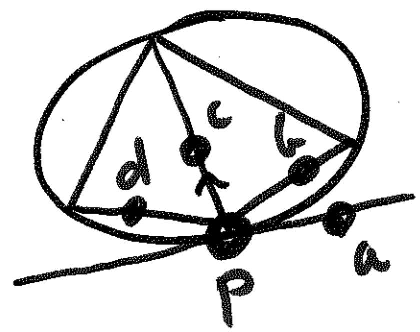
dim PGL(3, ℝ)

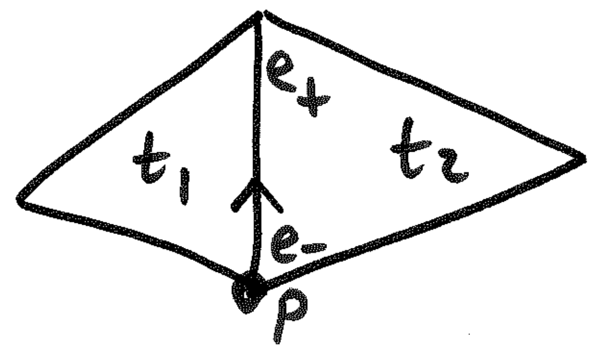
$\Omega \subset \mathbb{RP}^2$

$\circlearrowleft \Gamma \subset PGL(3,\mathbb{R})$

$$F = \Omega / \Gamma$$

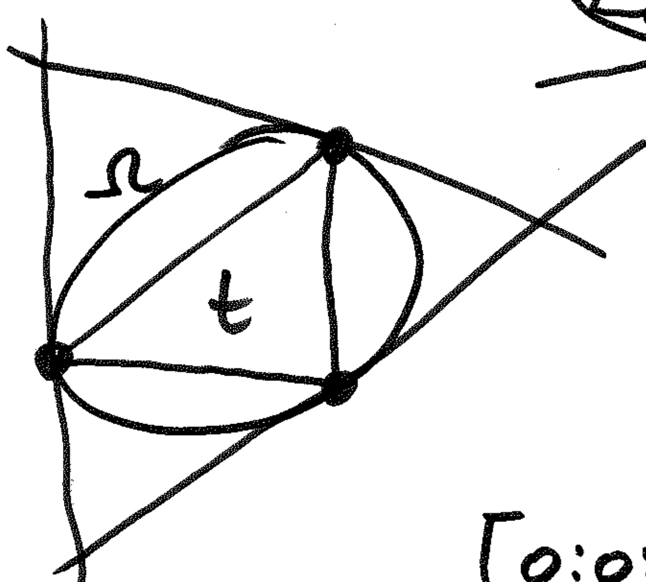
$e_- = CR(a,b,c,d)$

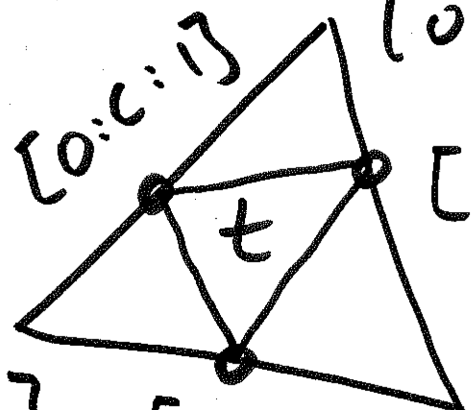




F.G.

$2^{\#punc} : 1$





$t = abc$