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Introduction to F(uzz) Theory

Heckman–Verlinde : paper from Wednesday

What 4d $\mathcal{N}=1$ SYM can be obtained from F-theory in a <u>strict decoupling limit</u> ?

$$\begin{array}{ccc} S = \text{surface in} & CY_4 & \leftarrow \mathbb{E}_c \\ = dP_n & \downarrow & \\ & S \subseteq B_3 & \end{array}$$

gauge degrees of freedom arise from 7-branes wrapping $S \times \mathbb{R}^{3,1}$

7-brane theory $\simeq$ 8d $\mathcal{N}=1$ SYM

$\hookrightarrow$ 4d $\mathcal{N}=1$ SYM + KK modes
+ ints with closed strings

Decoupling limit

- $\mathrm{Vol}_{\text{closed}}(S) \to 0$
- Zero slope limit
  $\alpha' \to 0$; focus on energies below string excitations

$$\frac{4\pi}{g_{YM}^2} = \frac{\mathrm{Vol}_{open}(S)}{(2\pi)^2 g_s}$$

open string metric G
closed string metric g $\Big\} \to$ $G_{open} = g_{closed} - 2\pi\alpha' B g^{-1} B + \cdots$ in small B expansion

or $G^{ij} = \left(\frac{1}{g + 2\pi\alpha' B} \, g \, \frac{1}{g - 2\pi\alpha' B} \right)^{ij}$

DBI:

$$\mathcal{L}^{open}_{DBI} = \mathcal{L}^{closed}_{DBI} + O(B^2)$$

$$\mathrm{Vol}_{open}(S) = \mathrm{Vol}_{closed}(S) + \int_B \mathcal{B} \wedge \mathcal{B}$$

$$\mathcal{B} = F + \pi^* B$$

$\uparrow$

$p B_{NSNS} + q B_{RR}$

Seiberg-Witten:
get NC gauge theory

$S = dP_n$ fuzzy

$(z_i, \bar{z}_i) \to (z_i, z_i^\dagger)$

$$[z_i, z_j^\dagger] = \theta_{ij}\,, \qquad \theta_{ij} = \frac{1}{B_{ij}}$$

— the KK tower gets truncated $\qquad dz_i \quad c_i$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad d\bar{z}_i \quad c_i^\dagger$

— S collection of finite # of fuzzy points

Fock space of $z_i, z_i^\dagger$

$$\dim S = \dim \mathcal{F}_S = \frac{1}{(2\pi)^2}\int B \wedge B$$

"Curly B's"

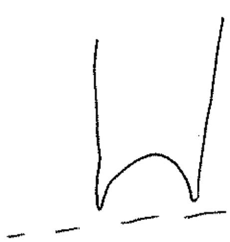
quantization condition: $\displaystyle\int_{\Sigma \subset S} B \in 2\pi\mathbb{Z}$

- $\int B \leftrightarrow$ D5-branes
- $\int B \wedge B \leftrightarrow$ D3-branes

} dissolved in 7-branes

$$\Rightarrow \frac{4\pi}{g_{YM}^2} = \frac{1}{g_s}(\#\text{ pts in } S)$$

$\tau = C_0 + i\, g_s^{-1}$



$g_s \simeq 1$

Fuzz - theory

1. NC geometry for fuzz branes
2. F vs. Fuzz

Toric examples (GLSM)

$\mathbb{F}_C$

$\downarrow$

$\mathcal{O}_{z}B_3$

$f(z, z_i)$ $\cancel{f(z, z_i, z_i^\dagger)}$

$g(z, z_i)$ $\cancel{g(z, z_i, z_i^\dagger)}$

GLSM	NC
$z_1, \dots, z_r$ chiral superfields	z_i
$U(1)^s$	$[z_i, z_j^\dagger] = \hbar \delta_{ij}$
$\vec{q}_i$ under $U(1)^s$	$[z_i, z_j] = 0$
$\sum \vec{q}_i \|z_i\|^2 = \vec{h}$	$\vec{D} = \sum \vec{q}_i z_i^\dagger z_i$
$X(\vec{h}) = \mathbb{C}^r / U(1)^s$	$\mathcal{F}(\mathbb{C}^r) = \langle \prod_i (z_i^{\dagger n_i}) \|0\rangle \rangle$
$\dim_{\mathbb{C}} X(\vec{h}) = r - s.$	$= \bigoplus_{\vec{h}} \mathcal{F}_X(\vec{h})$

$$\vec{D}\, \mathcal{F}_X(\vec{z}) = Z\, \mathcal{F}_X(\vec{z})$$

$$\{c_i, c_j^\dagger\} = \hbar\, \delta_{ij}$$

dz_j $\quad$ $d\bar{z}_j$

$$\mathcal{F}(T^*\mathbb{C}^r) = \left\langle \prod (z_i^\dagger)^{n_i} (c_i^\dagger)^{m_i} |0\rangle \right\rangle$$

$\Rightarrow$ differential forms

$\Rightarrow$ line bundles

$\Rightarrow$ subvarieties

F-theory	Fuzz-theory
$y^2 = x^3 + fx + g$	$f(z, z_i),\ g(z, z_i)$
$\Delta = 4f^3 + 27g^2 = 0$	$X = \mathbb{C}^r / U(1)^s$
$SU(5)$	$S = P_1 \cap \cdots \cap P_{r-s-2}$
$\Delta = z^5 \tilde{\Delta}$	
section N_{S/B_3}	$\Delta = z^5 \tilde{\Delta}(z_i, z)$

z $\quad$ S

F-theory

$8d$ $\mathcal{N}=1$ SYM.

$S \times \mathbb{R}^{3,1}$

partially twisted:

$U(1)_R \times U(2) \supset U(1)$

isometry of S

$\Rightarrow Q_\alpha, \bar{Q}_{\dot\alpha}$

V = vector multiplet

A = adj chiral $(0,1)$-form on S

Φ = adj chiral, $(2,0)$-form on S

$$\mathcal{L} = \int d^4\theta\, K + \int d^2\theta\, W + h.c. + \cancel{W^2 W}$$

$$K_\Phi = (\bar\Phi e^{-V}, \Phi e^{V})$$

$$K_A = (\bar\partial + A)^\dagger e^{-V}, (\bar\partial + A) e^{V}) - (\bar\partial^\dagger e^{-V}, \partial e^{V})$$

$$W = \mathrm{Tr}(\Phi \wedge (\bar\partial A + A \wedge A))$$

$$J \wedge F_S + [\Phi^\dagger, \Phi] = 0$$

$$\bar\partial_A \Phi = \partial_A \bar\Phi = 0$$

Fuzzy-theory

$\rightarrow V(z_\lambda^\dagger, z_\lambda)$

$\rightarrow \sum A_i(z_\lambda^\dagger, z_\lambda) C_i^\dagger$

$\rightarrow \sum \Phi_{ij}(\,,\,) C_i C_j$

$$\begin{array}{c} G_5 \\ \downarrow \\ G_2 \end{array}$$

$$\Psi^{(\vec{Q})} = \sum_{\vec{v}-\vec{u}=\vec{Q}} \Psi_{I,\vec{u},\vec{v}} \, \bar{\sigma}_{I(\vec{u})}(z^{+}) \, \sigma_{\Xi(\vec{v})}(z)$$

basis of ops $[D,\sigma] = \vec{u}\,\sigma$

$\vec{u} = 0 \longleftrightarrow$ zero modes

$\vec{u} \neq 0$ (K-K modes)

$$\sum_{\mu \in \mathcal{F}(\vec{G})} \langle \mu | \, ((\chi^{(Q)})^{+}, \chi^{(Q)}) \, | \mu \rangle$$

bracket in open string metric

For large $\vec{v}$, small $\vec{G}$ this truncates KK expansion become finite series.

$\mathrm{Vol}_{open}(S) = \int B \wedge B$, induced D3-brane charge

$\boxed{N_c \times N \text{ D3 stuck}}$ large N limit?

M_{KK}^{open}

$\left\{ \begin{array}{l} N^2 \text{ light vector mult.} \\ N^2 \text{ Scalar mult.} \end{array} \right\}$ D3 bran $SU(N_c)^N$

4d $SU(N_c)$ 7-brane
ADE

$N \sim \frac{1}{\alpha_{GUT}} \sim 25$