Morrison F-theory, II

$$\begin{tikzcd} X \arrow[d] \arrow[drr] & & \mathcal{L} \arrow[d] \\ \bar{X} = \{ y^2 = x^3 + f x + g \} \arrow[rr] & & B \end{tikzcd}$$

$\mathcal{L}$ line bdl

$f \in H^0(\mathcal{L}^{\otimes 4})$, $g \in H^0(\mathcal{L}^{\otimes 6})$
$x \in H^0(\mathcal{L}^{\otimes 2})$, $y \in H^0(\mathcal{L}^{\otimes 3})$

$\Delta = 4f^3 + 27g^2$

- $(\Delta = 0) = \Sigma\, m_i \Sigma_i$
  $\operatorname{ord}_{\Sigma_i}(\Delta) = m_i$
- $\Sigma_i = (s_i = 0)$ locally
- $\Delta / s_i^n \big|_{\Sigma_i} = \alpha^k \big|_{\Sigma_i} \cdot \beta^d \big|_{\Sigma_i}$

Group	RDP	sing f_0	ord f	ord g	ord Δ	β	α
$SU(n)$, $Sp(\lfloor \frac{n}{2} \rfloor)$	A_{n-1}	I_n, $n \geq 1$	0	0	n	g/f	$\frac{(\Delta / s_i^n)}{\beta^2}$
—	—	II	1	1	2	1	1
$SU(2)$	A_1	III	1	≥ 2	3	1	f/s_i
$SU(3)$, $Sp(1)$	A_2	IV	≥ 2	≥ 2	4	g/s_i^2	1
$SO(8)$, $SO(7)$, G_2	D_4	I_0^*	≥ 2	≥ 3	6	[special]	
$SO(2n+8)$, $SO(2n+7)$	D_{4+n}	I_n^*, $n \geq 1$	2	3	$n+6$	$\Delta/(s_i f)^3$; $3w^2 \equiv n \bmod 2$	$w^2 \left(g/(f s_i) \right)$
E_6 / F_4	E_6	IV^*	≥ 3	4	8	g/s_i^4	1
E_7	E_7	III^*	3	≥ 5	9	1	f/s_i^3
E_8	E_8	II^*	≥ 4	≥ 5	10	1	g/s_i^5
non-min			≥ 4	≥ 6	≥ 12	—	—

G = associated group

$G \longrightarrow Gl_n = \bigtimes_{i=1}^{k} GL_{n_i}$

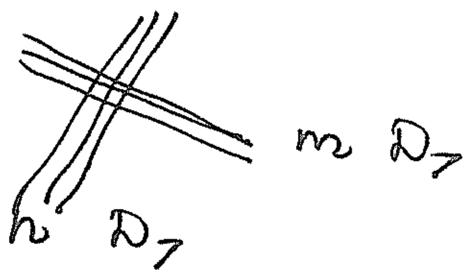
Any fibre on Σ_i which is worse than generic contributes an irrep. (usually)

$h^{1,0}(\Sigma_i)$ or $h^{1,0}(\tilde{\Sigma}_i)$ also contributes

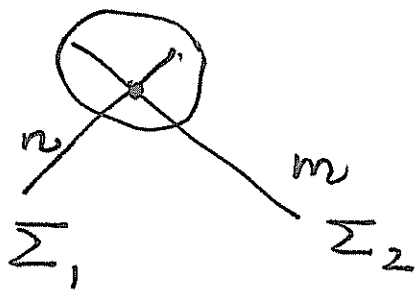
- if there is monodromy, $\tilde{\Sigma}_i$ is monodromy cover

<u>Example for physicists:</u>

$SU(n) \longleftrightarrow I_n \longleftrightarrow n\ D_7$ branes



bifundamental of $SU(n) \times SU(m)$



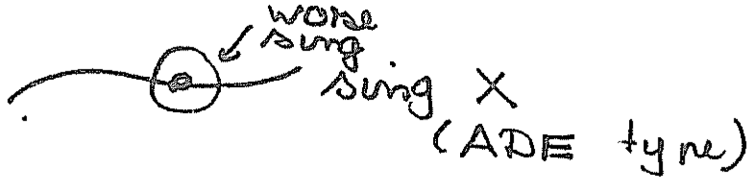
<u>2 Methods for determining reps.:</u>

B
$\cup\!|$
Σ_i codim 1
$\cup\!|$
Π_α codim 2

1) <u>Katz-Vafa</u>

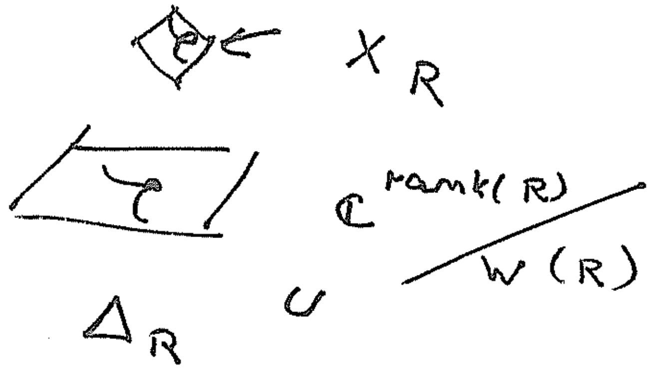
exclude places where non-minimal line on Kodaira table is reached in codim 2.

Fibers over Γ_a are still of ADE type, i.e., the total space is ADE



$\Gamma \longleftrightarrow$ its ADE type deforms to the Σ type

Γ: type R



$$\mathbb{C}^{\text{rank}(R)} \supseteq \bigcup_{\substack{\rho \text{ root} \\ \text{in } R}} \rho^{\perp}$$

$$\mathbb{C}^{\text{rank}(R)} / W(R) \supseteq \Delta = \left(\bigcup \rho^{\perp}\right) / W(R)$$

<u>Example</u>: $R = A_{n-1}$

$xy = z^n$ singularity

$xy = z^n + a_2 z^{n-2} + a_3 z^{n-3} + \cdots + a_n$

$a_2, \ldots, a_n$ = elt sym. fcts of ...

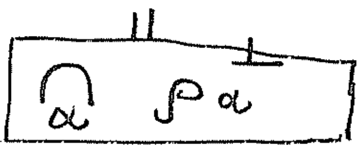
such that $\sum r_i = 0$

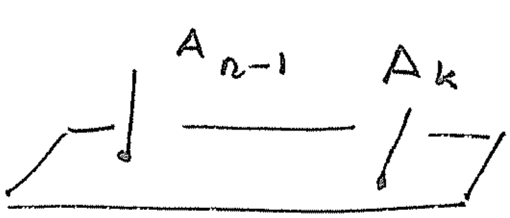
$$x\, y = \sum_{j=1}^{n} (z - r_j)$$

$$\mathbb{C}^{n-1} = \{ (r_1, \ldots, r_n) \mid r_1 + \ldots + r_n = 0 \}$$
$$\cup\!\!\mid$$
$$\{ r_i = r_j \}$$

$$\text{disc}\,(z^n + a_2 z^{n-2} + \ldots + a_n)$$
$$= \text{image of } \cup \{ r_i = r_j \}$$
$$\text{in } \mathbb{C}^{n-1}/S_n$$

$$R \supseteq R'$$





$k < n-1$

$$G_R \supseteq G_{R'}$$

decompose adjoint rep of G_R under $G_{R'}$
$= \text{adj}\, G_{R'} \oplus \rho$

<u>Def</u>: Assign ρ to $\sqcap$ (usually)

$A_n \supseteq A_{n-1}$ ($SU(n)$) $\quad \rho = \text{fund rep} \oplus \overline{(\ )}$

D_n ($SO(2n)$) $\supseteq A_{n-1}$ ($SU(n)$) $\quad \rho = \Lambda^2(\text{fund. rep})$

$E_n \supseteq A_{n-1}$, $n = 6, 7, 8$ $\quad \rho = \Lambda^3(\text{fund. rep})$

- $D_{n+1} \supseteq D_n$ ρ = vector rep of $SO(2n)$

  $SO(2n+2)$ $SO(2n)$

- $E_{n+1} \supseteq D_n$ ρ = spinor rep of $SO(2n)$

- $E_{n+1} \supseteq E_n$ E_6 : $-$ 27 dim'l

  E_7 : $-\frac{1}{2}$ (56 dim'l)

$$\tilde{\Sigma}_i \downarrow \Sigma_i$$

branch pts
or
zeros of β

if there is no monodromy we also get $h^{1,0}(\Sigma_i)$ copies of adj G_i

$\supseteq$ $\tilde{G}_i(\tilde{g}_i) = g_i$

$h^{1,0}(\tilde{\Sigma}_i) \leftrightarrow$ adj $\tilde{G}_i \setminus$ adj $G_i \leftrightarrow \rho$ assoc. to branch pts.

$h^{1,0}(\Sigma_i) \leftrightarrow$ adj G_i

<u>outline:</u>

define 2 or 3 basic reps for each grp (line in Kodaira table)
w/ geometric description of location

<u>Conj</u>: All reps are built out of these as geometric condition coalesce

$$h^{1,0}(\Sigma_i) \longleftrightarrow \mathrm{adj}\, G_i$$

$$\tfrac{1}{2}\, \cancel{h^{1,0}}\, (\beta = 0) \longleftrightarrow \rho_\beta(\mathrm{adj}\, \tilde{G}_i \setminus \mathrm{adj}\, G_i)$$

virtual

$$\alpha = 0 \longleftrightarrow \rho_\alpha$$

$$\tfrac{1}{2}(K_\Sigma + \Sigma)\big|_\Sigma = g-1$$

<u>dim $B=2$</u>:

if we avoid non-minimal line then there must be a formula in Alg. Cycles on B $A^*(B)$ $\otimes$ (diff forms on $\mathbb{R}^6$)

$$\mathrm{tr}(R^2),\ \mathrm{tr}(F^2),\ \mathrm{tr}\, F^4,\ \mathrm{tr}\, R^4$$

$$\deg\left(\tfrac{1}{2}\left(\tfrac{1}{2} K_B\, \mathrm{tr}\, R^2 + 2 \sum \Sigma_i\, \mathrm{tr}\, F_i^2\right)^2\right)$$

$$= \frac{K_B^2}{8}(\mathrm{tr}\, R^2)^2 + \frac{1}{6}\mathrm{tr}\, R^2 \sum(\mathrm{Tr}_{adj} F_i^2 \ldots$$

$$- \sum n_\rho\, \mathrm{tr}_\rho F_i^2) - \frac{2}{3}\sum(\mathrm{Tr}_{adj} F_i^4 - \sum_\rho n_\rho \mathrm{Tr}_\rho F^4)$$

$$+ 4 \sum_{i<j} n_{\rho\sigma} \, \mathrm{Tr}_\rho F_i^2 \, \mathrm{Tr}_\sigma F_j^2$$

$$+ \underbrace{(\cdot\cdot)}_{= (H - V + 29T)} \, \mathrm{tr}\, R^4 \; \ldots$$

$$\rho_\beta \, \frac{1}{2} \, \Gamma_{\beta, i} + \rho_\alpha F_{\alpha, i} + (\mathrm{ad}_j) \, \frac{1}{2} \left(\cancel{X}_{\Sigma_i} + \Sigma_i \right) \Big|_{\Sigma_i}$$