

Morrison: "Much ado about  $N=2$ "

Recent developments in  $N=2$  SUSY  
4d gauge theory:

gauge theory via lagrangian:

input data: compact semi-simple grp  $G$   
+ finite dim. rep. of  $G$  on  $V$   
 $\rho$

$\mathcal{L}_{G,\rho}$  = kinetic term of gauge fields,  
interaction terms, ...  
coupling  $\tau \in \mathfrak{h}_g$   $\left\{ \begin{array}{l} G \\ V_H \end{array} \right.$

$u = \text{tr}(|\psi|^2)$

$\leadsto$  classical gauge theory

renormalization changes  $\tau \dots$

unless the theory is conformal

example

$G = SU(2); \rho: SU(2) \hookrightarrow SU(8)$   
 $U(1) \searrow$   
 $SU(2)^4$

$N_f = 4 \quad \underbrace{\mathbb{C}^2 \oplus \dots \oplus \mathbb{C}^2}_4$

in general:  $G = SU(N)$

conformal  $\iff N_f = 2N$

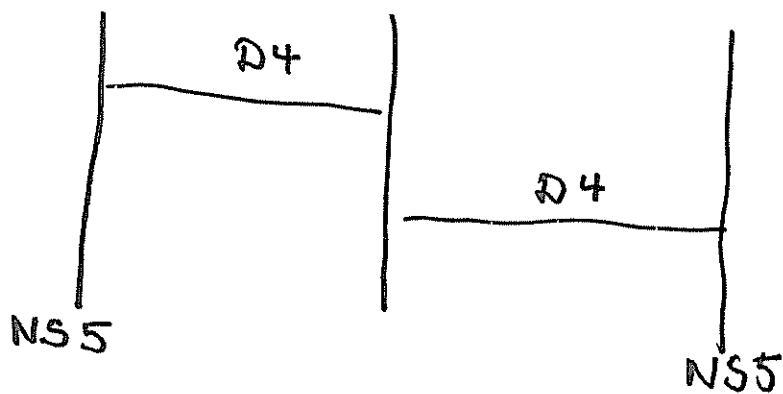
Seiberg - Witten  
SU(2)

$$\tau \in \mathfrak{h}_g / \text{SL}(2; \mathbb{Z}) \subseteq \mathbb{CP}^1$$

1 missing pt.

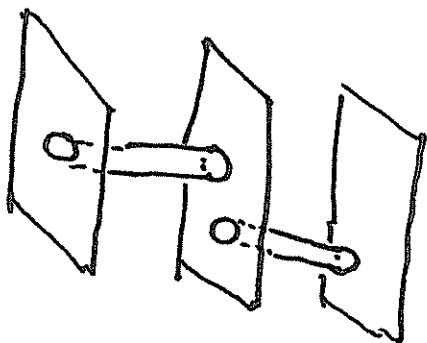
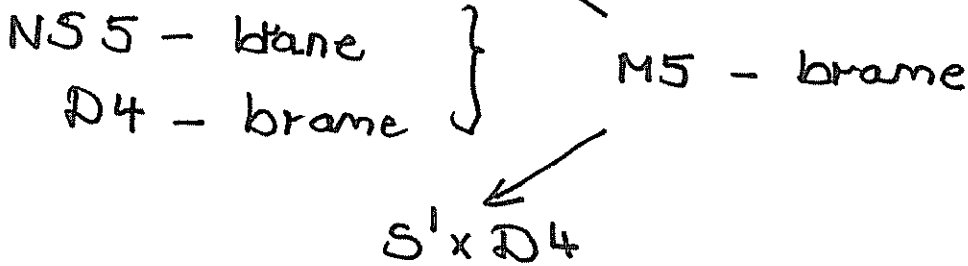
$$\tau \rightarrow i\infty \quad \text{weak coupling description}$$

string inspired construction:



Type II A  
 ↑  
 M-theory

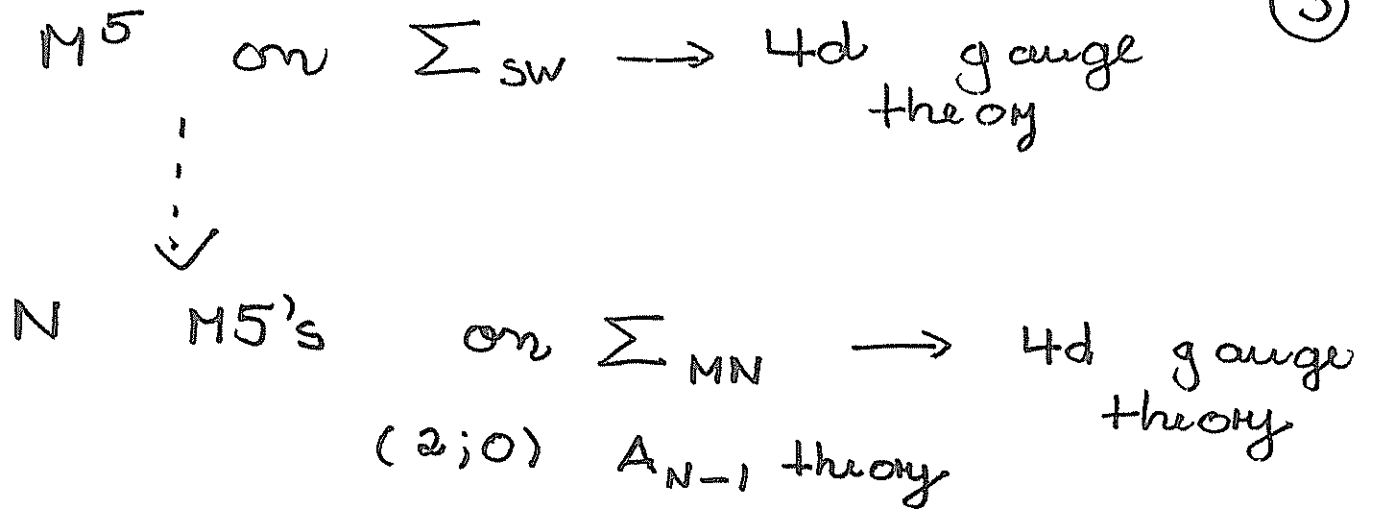
unwrapped



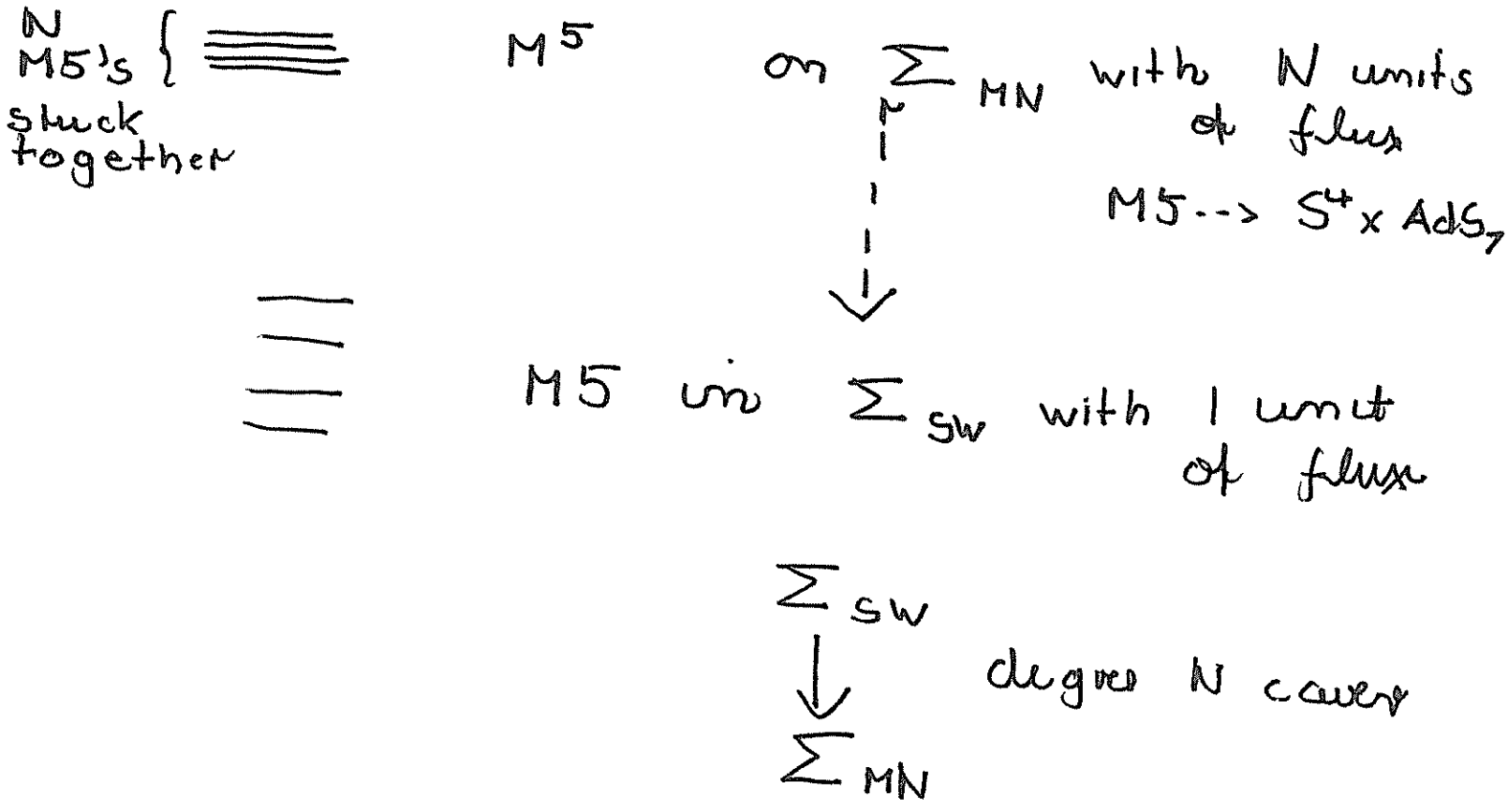
$\Sigma \subset \parallel \text{dbrane}$

Construction technique:

$$M^5 \text{ on } \Sigma \rightarrow 4d \text{ gauge theory}$$



interpolating sugra solution:



$G = sw(\mathbb{C}P^1), N_{\pm} = 4$

$\Sigma_{MN} = \mathbb{C}P^1 \setminus \{4 \text{ pts}\}$

$\tau / sl(2; \mathbb{Z}) =$  cross ratio of 4 pts

$SU(3)$ ,  $N_f = 6$  (Aiyas, Seiberg) (4)

$\tau \in \mathfrak{h}_g / \Gamma_0(2) = \mathbb{C}P^1 \setminus 2 \text{ pts}$

one pt: weakly coupled theory  
 other pt: ?

$\tau \rightarrow 1 \Rightarrow$  weakly coupled  $SU(2)$   
 coupled to  $E_6$  global symmetry theory

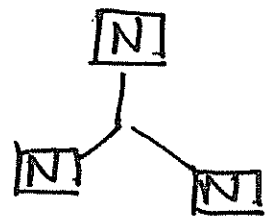
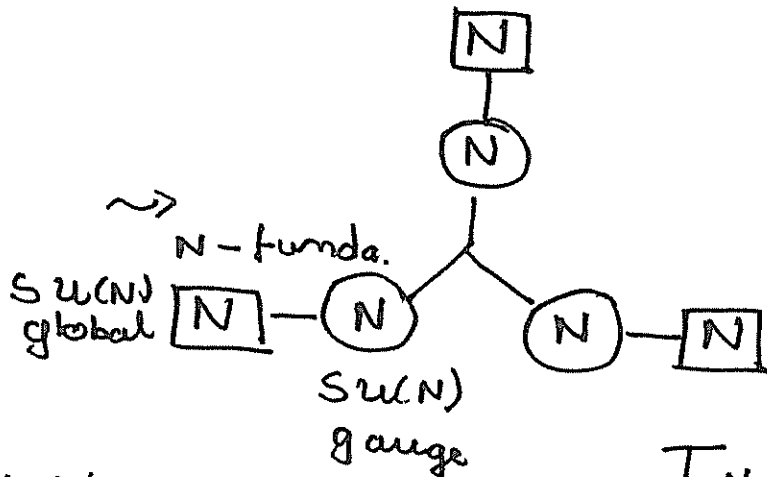
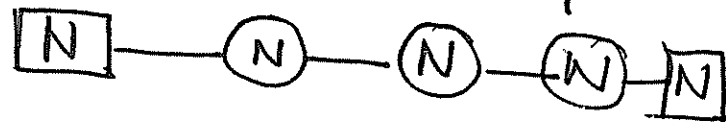
Gaiotto (0904):



global  $SU(N)^3$  symmetry  
 trifundamental field

in general

take  $G = \prod G_a$   
 semi-simple



$T_N$

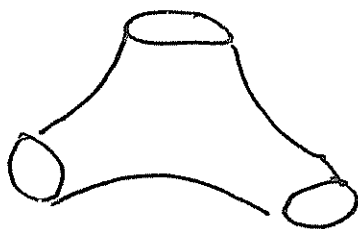
Picture:

$T_2$ : theory of  $SU(2)^3$  hypers  $\rightarrow SO(8)$

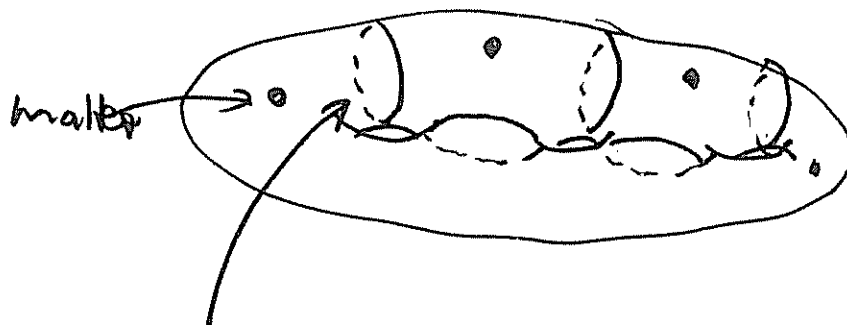
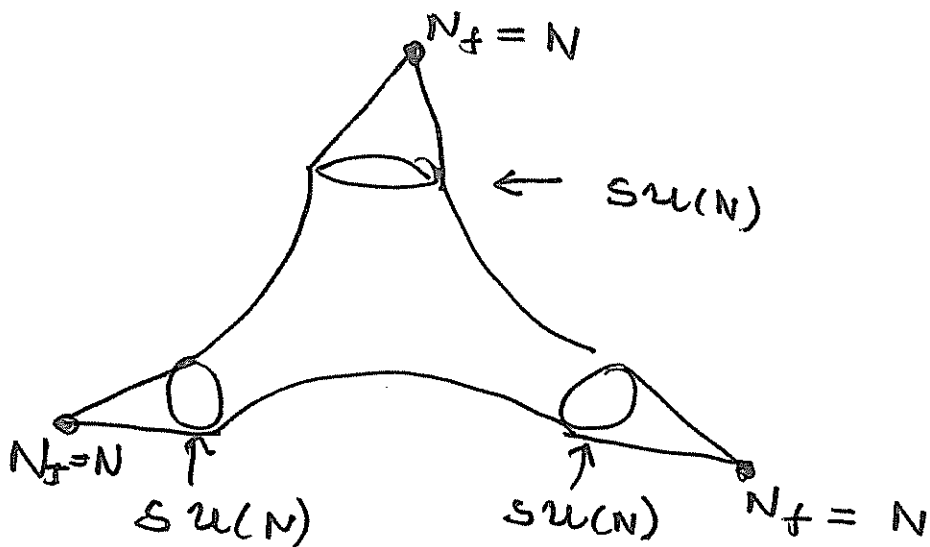
$T_3$ :  $E_6$  global sym. enhanced to  $SU(3)^3$

Picture.

$N$   $M^5$  on



$\leftrightarrow$  junction  
in field  
theory



$SU(N)^6$  with



+ matter at picture

moduli = moduli of Riemann surfaces genus  $g$ ,  
 $n$  punctures,  ~~$3g-3+n$~~   
 $\chi(\Sigma_{g,n}) < 0$

Two other directions:

1) Alday - Gaiotto - Tachikawa:

Nekrasov's partition fct for  
 4d theory which is  $\mathbb{D}$  M5 on

$$\Sigma_{g,n}$$

$$\updownarrow$$

conformal blocks in Liouville theory  
 on  $\Sigma_{g,n}$

$\rightarrow \parallel$

2) Loop operators:

- Gaiotto - Moore - Neitzke  
 calculated loop ops from  $\Sigma_{sw}$
- Druker - DM - Aldaya:  
 calculated loop ops ( $Sw(2)$  on  $\Sigma_{MN}$ )

# 1-1 correspondence:

