21 November 2008
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Quiver representations and D0/D2/D6 bound states

Introduction

Math	Physics
	$\mathcal{N}=1$ SQM, X Kähler
$\chi(X)$	$\mathrm{Tr}\,(-1)^F$
	$Q_5 : \psi \to \bar{\psi}$
$\tau(X)$	$\mathrm{Tr}\,Q_5$
	$\mathcal{N}=\frac{1}{2}$ SQM on X
$\mathrm{ind}(\partial)$	$\mathrm{Tr}'\,(-1)^F$
	$\mathrm{Tr}\,(-1)^F e^{-\beta H}$

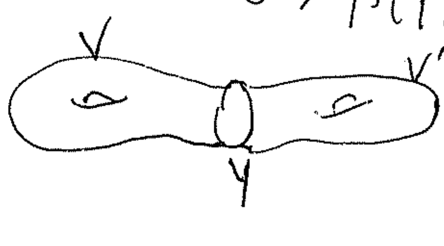
$$\mathrm{ind}(\partial) = \int_X \hat{A}(X)$$

TQFT on X

$\rightsquigarrow Z(X)$

TQFT on $\mathbb{R} \times Y^{(3)}$

$\rightsquigarrow \mathcal{H}(Y)$



$v \in \mathcal{H}(Y)$
$v' \in \mathcal{H}(Y)^*$
$Z(X) = \langle v', v \rangle.$

boundaries / boundaries $\rightsquigarrow$ Category

B-model on X : X is Kähler

$$\Phi : \Sigma \to (X, g)$$

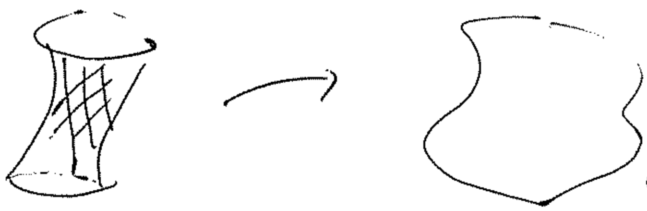
$$K_\Sigma = \Lambda^n T_\Sigma^*$$

$$\Psi_+^I \in \Gamma(K_\Sigma^{1/2} \otimes \Phi^*(TX))$$

$$\Psi_-^I \in \Gamma(\bar{K}_\Sigma^{1/2} \otimes \Phi^*(TX))$$

Q-cohomology

$$H^p(X, \Lambda^q T^{1,0} X)$$

Σ  CY

Boundary Chan-Paton

$$\int_{\partial\Sigma} A_\mu \, d\phi^\mu - 2i F_{\mu\nu} \Psi_+^\mu \Psi_-^\nu$$

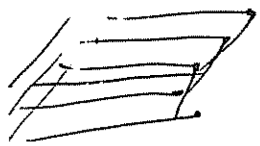
and then topologically twist

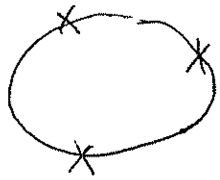
Massless states

$$H^n(X, \mathcal{E}^\vee \otimes \mathcal{F})$$

Superpotentials

N D3-branes in type IIB





$$W = \mathrm{Tr}(XYZ - XZX)$$

$$W = \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1) \quad \text{conifold}$$

(1) Klebanov - Witten (symmetry arguments)

(2) Morrison - Plesser: $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$, Douglas-Moore

(3) Aspinwall - Katz: sheaf

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Closed string B-model

WDVV relations '91

3 pt function

$\leftarrow$ descendant

$$C_{ijk}(t) = \langle \phi_i \phi_j \phi_k \, e^{\sum t_\ell \phi_\ell^{(1)}} \rangle$$

$\nwarrow$ moduli of the CY

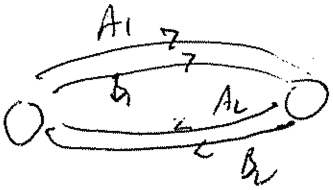
$$\partial_i C_{jkl}(t) = \partial_j C_{ikl}(t)$$

$$\Rightarrow \quad C_{ijk}(t) = \partial_i \partial_k \partial_l \mathcal{F}(t)$$, WDVV potential.

tt^* special geometry

<u>Open strings</u>

A^∞ algebra



<u>Math introduction:</u>

<u>Quiver representations</u>

- No superpotential

$$\begin{array}{ccccc} e_0 & \xrightarrow[\sigma]{f} & e_1 & \xrightarrow[\tau]{g} & e_2 \\ V_0 & & V_1 & & V_2 \end{array}$$

Path algebra kQ $\qquad$ k – arbitrary field, $k = \mathbb{C}$ for physicists

basis of paths $\left\{ \begin{array}{l} \left. \begin{array}{l} e_0 \\ e_1 \\ e_2 \end{array} \right\} \text{null paths} \\ \sigma \\ \tau \end{array} \right.$

$\sigma\tau$ = a path

$\tau\sigma = 0$.

<u>Example</u> $\bullet \longrightarrow \bullet$

rep is $A : V \to W$.

$$\underset{V}{\bullet} \xrightarrow{A} \underset{W}{\bullet} = \underset{\ker A}{\bullet} \longrightarrow \underset{0}{\bullet} \;\oplus\; \underset{\mathrm{Coim}\,A}{\bullet} \xrightarrow{\sim} \underset{\mathrm{Im}\,A}{\bullet} \;\oplus\; \underset{0}{\bullet} \longrightarrow \underset{W'' = \mathrm{Coker}}{\bullet}$$

$$= \left(\underset{\mathbb{C}}{\bullet} \longrightarrow \underset{0}{\bullet} \right)^{\oplus \dim(\ker A)} \oplus \left(\underset{\mathbb{C}}{\bullet} \longrightarrow \underset{\mathbb{C}}{\bullet} \right)^{\oplus \dim(\mathrm{im})} \oplus \left(\underset{0}{\bullet} \longrightarrow \underset{\mathbb{C}}{\bullet} \right)^{\oplus \dim(\mathrm{coker})}$$

Moduli space is 3 points.

<u>Example</u>: Kronecker Quiver

$$V \bullet \underset{\mu}{\overset{\lambda}{\rightrightarrows}} \bullet W$$

A rep: $\dim V = \dim W = 1$, λ, μ scalars

distinct reps if λ/μ differs

i.e., moduli space is $(\lambda : \mu) \in \mathbb{P}^1$

Fancier version: $D^b(\text{mod-}A) = D^b(\mathrm{Coh}\,\mathbb{P}^1)$

kQ - modules $\Longleftrightarrow$ representation of kQ.

<u>Example</u>

Path algebra: $kQ \cong \mathbb{C}[x]$

$\mathbb{C}[x]$ - modules $\longleftrightarrow$ Jordan canonical form.

$$\begin{pmatrix} \lambda & 1 & \\ & \lambda & 1 \\ & & \lambda \end{pmatrix}$$

Noncommutative geometry

X	A
$E \to X$	A-modules
X CY 3-fold	A CY algebra
$D^b(\mathrm{Coh}\, X)$	$D^b(A\text{-mod})$

Szendrői; Nagao-Nakajima

Disk amplitudes

<u>Theorem</u> (Segal + Bocklandt) Quiver with relations, CY algebra

$\Rightarrow$ quiver relations must be given by a superpotential

A $\mathbb{C}$-algebra A is <u>3-CY</u> if for all A-mods, M, N there is a perfect pairing

$$\mathrm{Ext}_A^i(M,N) \times \mathrm{Ext}_A^{3-i}(N,M) \to \mathbb{C}.$$