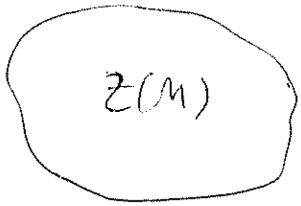
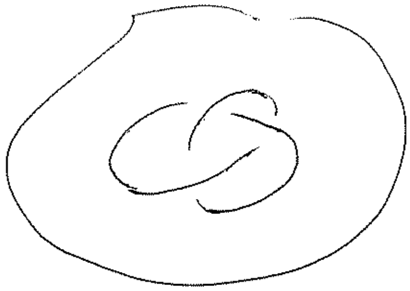
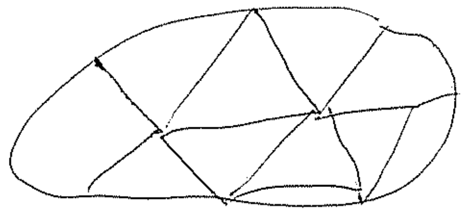
2 May 2008
T. Dimofte







G $\qquad\qquad U_q(\mathfrak{g})$

1. CS Theory
2. Geometric Quantization
3. Hyperbolic geometry / Hikami's invariant
4. Correspondence

$$I[A] = \frac{k}{4\pi}\int_M \mathrm{Tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

$$A \mapsto g^{-1}Ag + g^{-1}dg$$

$SU(2)$ $\qquad [T_a, T_b] = \varepsilon^{abc}T_c$

$$A = \sum_a A^a T_a$$

$2\pi \cdot k \cdot n$

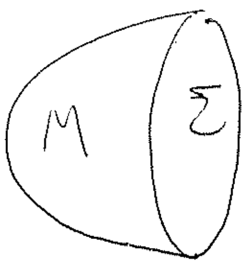
$$\int DA \, e^{-i I[A]}$$

$$F = dA + A \wedge A = 0$$

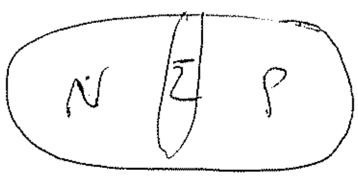
$$A = \omega + ie$$

$$\hbar = \frac{\pi}{k}.$$

$$Z_\hbar(M) = \sum_\alpha e^{\frac{1}{\hbar} S_0 - \frac{\delta}{2} \log \hbar + S_1 + \hbar S_2 + \cdots}$$

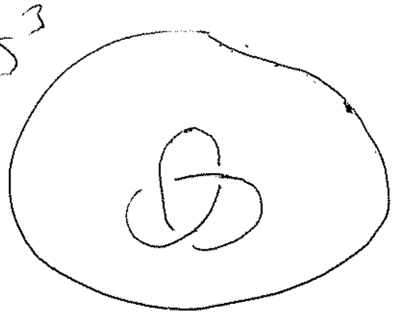


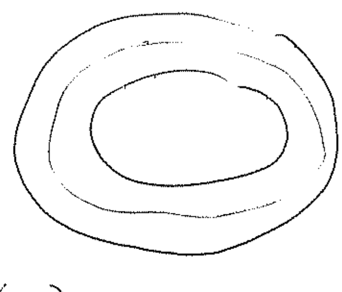
$Z(M)(\text{bdy}) \in \mathcal{H}_\Sigma$



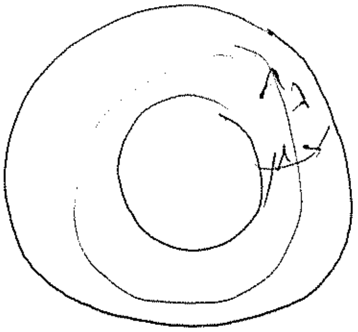
$= \langle Z(N) | Z(P) \rangle_{\mathcal{H}_\Sigma}$

S^3





$Z(M)$



$$\begin{pmatrix} l & * \\ 0 & 1/l \end{pmatrix} \begin{pmatrix} m & * \\ 0 & 1/m \end{pmatrix}$$

$l = e^v, \; m = e^u$

$\{u, v\} = 1$

$P = \{(u,v)\} = \mathbb{C}^* \times \mathbb{C}^*$

$\hat{u}, \hat{v} \qquad [\hat{u}, \hat{v}] = -i\hbar$

$\hat{u}\,\psi(u) = u\,\psi(u)$

$\hat{v}\,\psi(u) = i\hbar \frac{d}{du}\psi(u)$

$\mathcal{H}_{T^2} = L^2(\mathbb{C}^*)$

$(u_0, v_0) \in P$

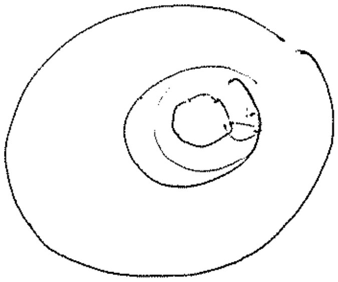
$f(u,v) = 0 \qquad \hat{f}(\hat{u}, \hat{v})\,\psi(u) = 0$

$p, q \qquad (\hat{p} - p_0)\,\psi(q) = 0 \implies \psi(q) = e^{i\frac{p_0}{\hbar}q}$

$\hat{p} = -i\hbar\frac{d}{dq}$

$(\hat{q} - q_0)\,\psi(q) = 0 \implies \psi(q) = \delta(q - q_0)$

$A(l,m)=0$



$A(\text{unknot}) = l-1$

$A(3_1)=A(\text{trefoil}) = (l-1)(1+m^6 l)$

$A(4_1)=A(\text{figure } 8) = (l-1)(1-(1-m^2-2m^4+m^6+m^8)l+l^2)$

$\hat{A}(\hat{l},\hat{m})\, Z(M)(u)=0$

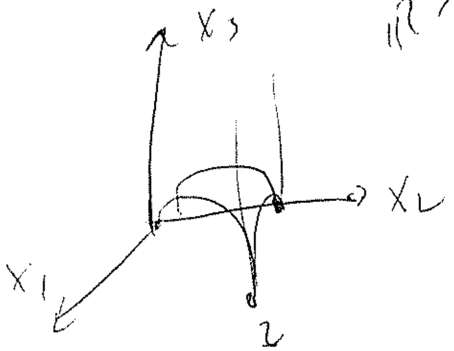
$\cancel{Z(M)}$

$Z(M;u) = e^{\frac{1}{\hbar}S(u)+\cdots}$

$\hat{l} = e^{i\hbar \frac{d}{du}}$

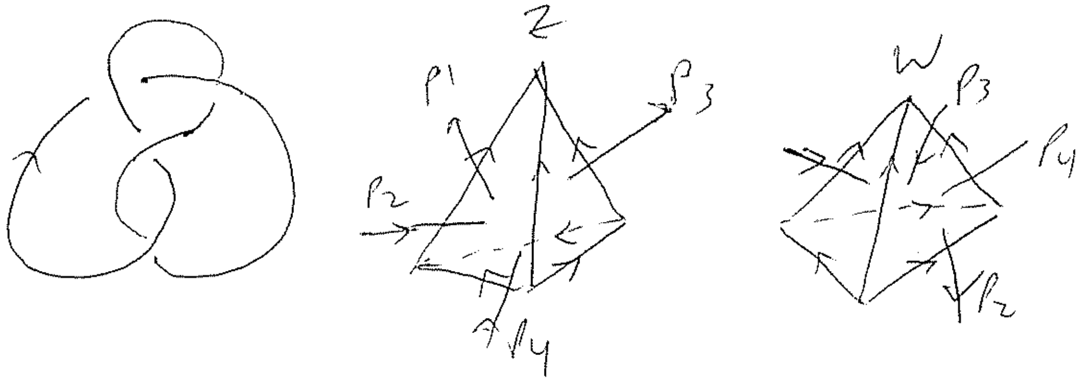
$A(e^{iS_0'(u)}, e^{u})=0$

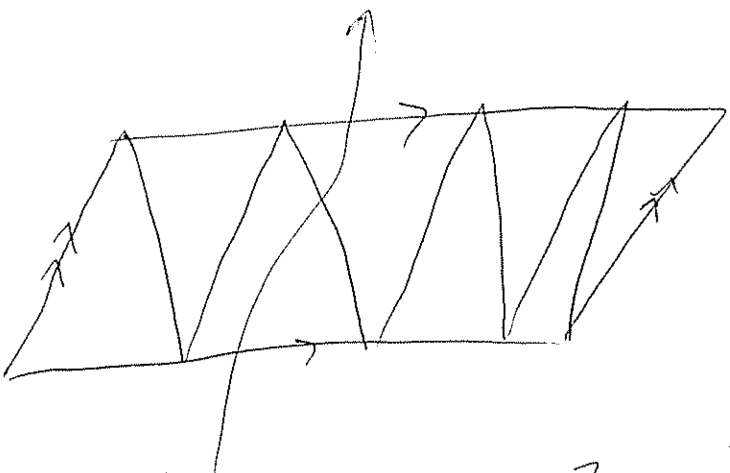
$S_0^{\text{geom}} = \frac{1}{2}\mathrm{Vol}(M,u)$



$ds^2 = \frac{1}{x_3^2}(dx_1^2+dx_2^2+dx_3^2)$

$PSL(2,\mathbb{C})$





holonomy mm

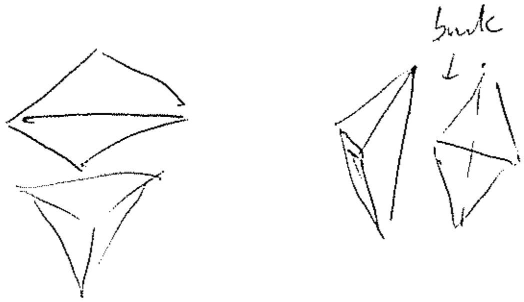
$r(z,w)=0$

$\frac{z}{w} = m^2 = e^{2u}$

$\triangle \sim V \otimes V \otimes V^* \otimes V^*$

$S_{ij} : V_i \otimes V_j \longrightarrow V_i \otimes V_j$

$S_{23} S_{12} = S_{12} S_{13} S_{23}$



$f(p)$

$f(p_1, p_2) \qquad S = e^{\frac{1}{i\hbar}\hat{q}_1 \hat{p}_2} \Phi_\hbar(\hat{p}_1 + \hat{q}_2 - \hat{p}_2)$

$$\hat{q}_i \sim 2\pi i \hbar \frac{d}{dp_i}$$

$$\Phi_\hbar(z) = e^{\frac{1}{4}\int_{\mathbb{R}+i\varepsilon} dx \, \frac{e^{-ixz}}{x \sinh(\pi x)\sinh(\hbar x)}}$$

In example: assign

$\langle p_1 p_3 | S | p_2 p_4 \rangle$, $\quad \langle p_4 p_2 | S^{-1} | p_3 p_1 \rangle$ to the two tetrahedra

$$\frac{\delta(p_1 + p_3 - p_2)}{\sqrt{4\pi k}} \Phi_\hbar(p_4 - p_3 + i(\pi + \hbar))$$

$$\delta(\quad) \frac{1}{\Phi_\hbar(\quad)}$$

$z = e^{p_4 - p_3} \qquad w = e^{p_1 - p_2}$

$$Z(M) = \int dp_i \prod \langle \quad \rangle \; \delta(\quad)$$

$$\delta(p_4 - p_3 - p_1 + p_2 = -u) \quad \left(\frac{z}{w}\right)^{-1}$$

$=$

$$= \frac{1}{4\pi\hbar} \int dp \, \frac{\Phi_\hbar(p + i\pi + i\hbar)}{\Phi_\hbar(-p - 2u - i\pi - \hbar)} \, e^{\frac{2i}{\hbar} u(u+p)}$$

$$\left[U_q(sl_2) \right]$$

$$\sim e^{\frac{1}{2\hbar} \mathrm{Vol}(M, u)}$$

$$\Phi_\hbar \sim e^{\frac{1}{2\hbar} \mathrm{Vol}(\Delta)}$$

$$L^2(\mathbb{R})$$

$$= \sum_{\text{saddle } \beta} e^{\frac{1}{\hbar} S_0^\beta - \frac{S_1^\beta}{2} \log \hbar + \cdots}$$

$$Z_\hbar^{CS\,\alpha}(u) = \frac{c}{\hbar} e^{-u} Z_\hbar^{Hik\,\alpha}(u)$$

$c = 2\pi$ for figure 8