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Further Aspects ---

(slide on Euclidean wallpaper)

(slideon the Hyperbolic Plane)

(slide on Escher hyperbolic wallpaper)

? Weeks manifold $\quad Vol = 0.9427$

Agol - Dunfield $\quad Vol > 0.67$

$$Л(\theta) = -\int_0^\theta \log|2\sin\theta|\, d\theta$$

Finite dimensional moduli space

G = Lie group, fin dim'l

X^n = homog. space

M = closed manifold

(G, X) Structure $\qquad M = X/\Gamma \leftarrow$ discrete

deformation space is open subset of alg. variety

$\rho_0 : \pi_1 M \longrightarrow \Gamma \subset G$ = matrix group / conjugacy

$g_1, \dots, g_n \longmapsto M_1, \dots, M_n$

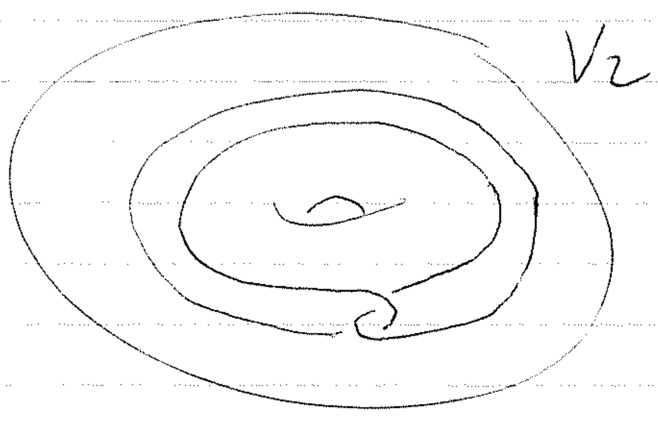
$\mathrm{Hom}(\pi_1 M, G) = \langle M_1, \dots, M_n,\ M_{\lambda_1}^{\pm} \cdots M_{\lambda_k}^{\pm} = I \rangle$

Whitehead 3-manifold

$W \neq \mathbb{R}^3$

$W \times \mathbb{R} = \mathbb{R}^4$

$W = \bigcup_{n>0} V_n \qquad S^1 \times \mathbb{R}^2$



V_2

Whitehead

V_1 V_2 $\mathbb{R}^3$

W NOT TAME

An n-manifold M is *tame* if there compact n-manifold N s.t. $M - \partial M = \text{interior}(N)$

$\mathbb{R}^3 = \text{int}(B^3)$ is tame.

Tameness Theorem (Agol, Caligari-Gabai)

If M^3 hyperbolic, $\pi_1 M$ is finitely generated then M is tame

Recognition Problem Solved

Can decide if 2 closed orientable 3-manifolds are the same or not.

(S^3)

J. Manning
Thompson
T. Li
Jaco
Rubenstein

dim 2 closed Compact manifolds, commutative monoid (add = connect sum)

$= \langle S, P, T : S \text{ is identity}, S + X = X$
$3P = P + T \rangle$

High dim's

$$S^7 = M_1 \# M_2 \quad \leftarrow \text{exotic 7-spheres}$$

$$= (M_1 \# M_2) \# \cdots \# (M_1 \# M_2)$$

Kneser (30's)

Closed 3-manifold has prime decomposition

prime not nontrivial connect sum

irreducible every sphere bounds a ball

irreducible $\Rightarrow$ prime

prime $\Rightarrow$ irreducible or $S^1 \times S^2$ or $S^1 \tilde{\times} S^2$.

Milnor (60)

Prime decomposition (closed M^3)

pieces are unique except

$$(S^1 \times S^2) + (S^1 \tilde{\times} S^2) = 2(S^1 \tilde{\times} S^2)$$

not orientable

Ex

(spheres to cut along are <u>not</u> unique)

<u>Torus decomposition</u> cut along tori into pieces which are either (1) SFS

or (2) hyperbolic

[or (3) Solv: orbifold bundle with generic fiber T^2]

aco halen ohannsen

<u>JSJ</u> use least number of tori

<u>Thm</u> M prime JSJ unique up to isotopy

<u>Graph manifolds</u>

<u>def</u> (Waldhausen 68)

$\bigcup$ (circle bundles)

glued along boundary tori

$S^1 \times D^2$, $S^1 \times P$ ← pants, $S^1 \tilde{\times} M$

Thm M^3 closed $\Rightarrow M = G \cup N$

graph tori, incompr. hyperbolic

G, N are unique.

If M is irreducible then get isotopy uniqueness

Thm Graph $\Leftrightarrow$ M has a Thurston decomposition
no hyperbolic pieces

$\Leftrightarrow$ M collapses with bounded curvature

Geometrization Conj/Thm (Thurston / Hamilton, Perelman, ---)

M closed orientable $\Rightarrow$ M has a prime decomposition into pieces, which have a JSJ decomposition into pieces, each of which has Thurston geometries.

Uniqueness of Geom? No.
of topology? Yes.

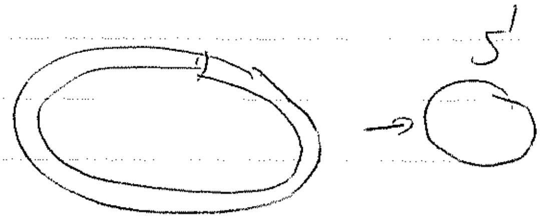
M^n collapses with bounded curvature

there is a sequence of Riem. metrics on M s.t.

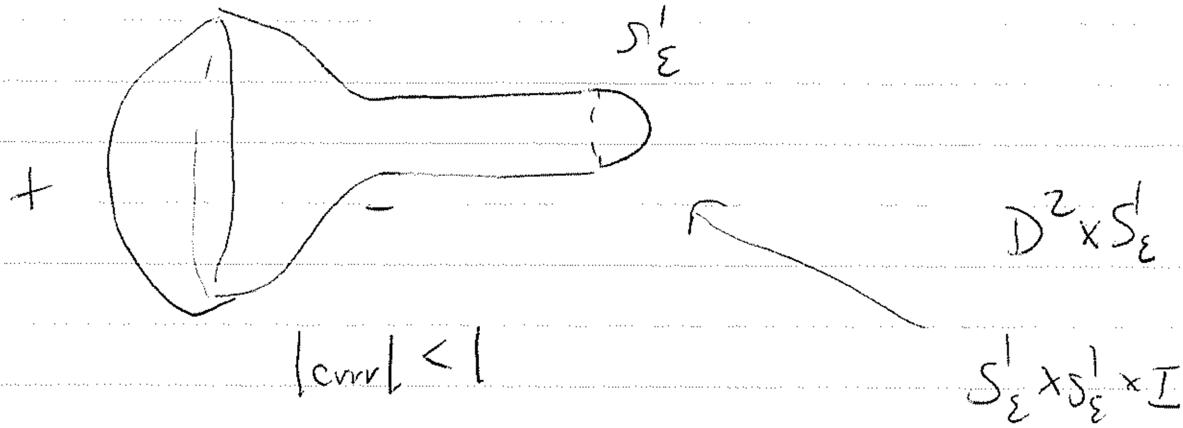
(1) $|\text{all curvatures}| \leq 1$

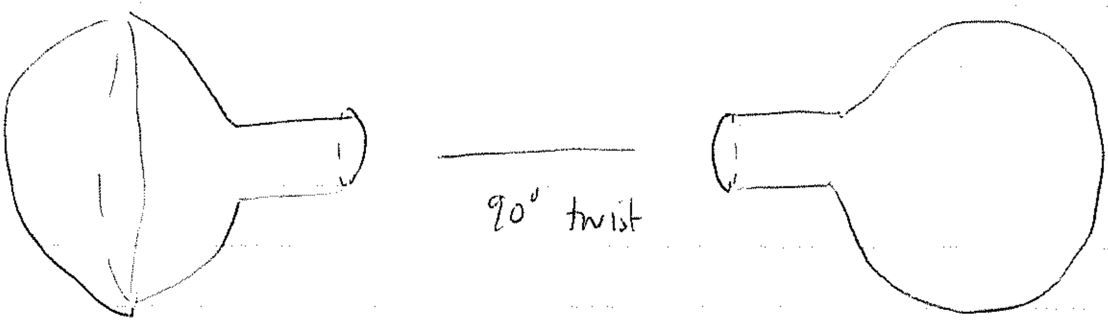
(2) $\forall x \in M$, $\text{inj}(x) \to 0$

Example Torus $S^1 \times S^1_\varepsilon$



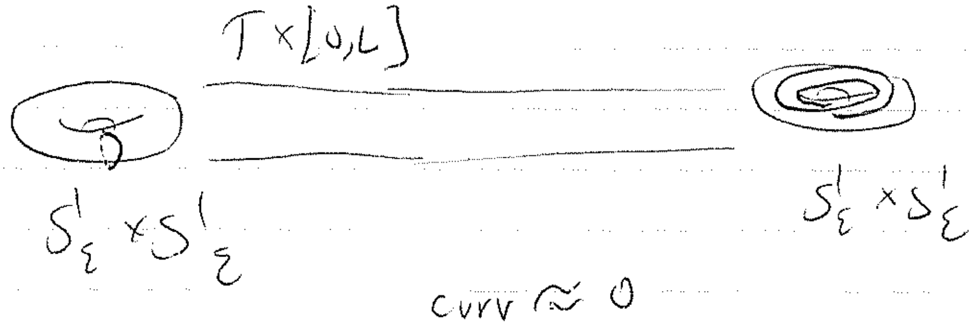
Example S^3 collapses





"genus 1 Heegard splitting of S^3"

$|\text{inj}| = \varepsilon$



<u>Cheeger-Gromov</u> (Collapsing thm)

If M^3 closed, collapses then it is graph.