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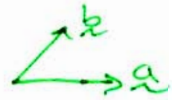
Notes by M. Gary

Geometry & Topology of 3-Manifolds

Euclidean Torus $T = \mathbb{R}^2 / (\mathbb{Z}\underset{\sim}{a} + \mathbb{Z}\underset{\sim}{b})$



area = 1

Marking (distinguish a & b)

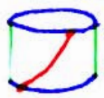
Moduli → Teichmuller Space $= \mathbb{H}^2 \underset{\text{topologically}}{\cong} \mathbb{R}^2$

Moduli → Moduli space = Mod surface

$= \mathbb{H}^2 / SL(2, \mathbb{Z}) =$ 

$\begin{pmatrix} \text{Automorphisms} \\ \text{of } T \end{pmatrix} \leftrightarrow GL(2, \mathbb{Z})$

Cutting open T along b

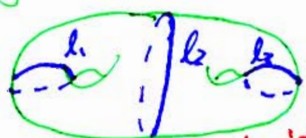


length(b) $\in (0, \infty)$

height $1/\text{length}(b)$

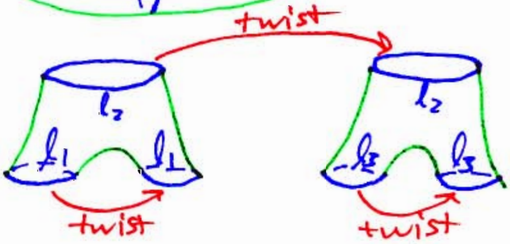
twist $\in (-\infty, \infty)$

Hyperbolic Surfaces



genus g

cut along 3g-3 curves to get pants decomposition

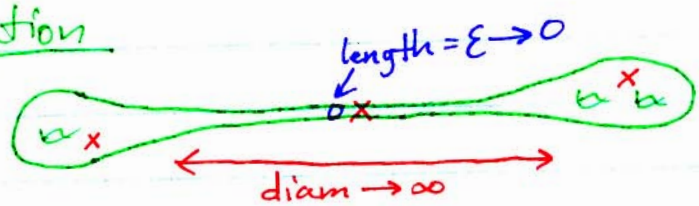


lengths determine pair of pants.

Marked $\longrightarrow \mathcal{T}(F_g) \cong \left((0,\infty) \times (-\infty, \infty)\right)^{3g-3} \cong \mathbb{R}^{6g-6}$

$\cong \mathbb{R}^{|\chi| \cdot \dim(SL(2,\mathbb{R}))}$ — $\text{Isom}(\mathbb{H}^2)$ $\qquad \chi = 2-2g$

Degeneration



Geodesic Lamination

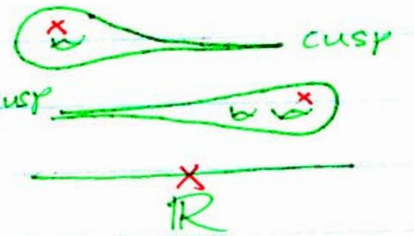
pinched "curve"

(Thurston Boundary S^{6g-7}

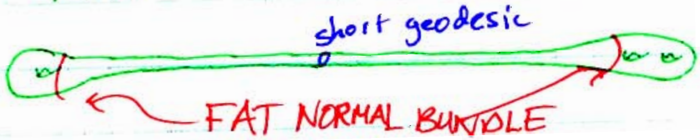
compactified space is Ball of dim $6g-6$

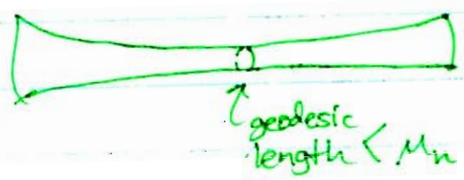
Take a base point $\times$

limit) Gromov Hausdorff



Margulis in a hyperbolic n-manifold short geodesics have FAT normal bundles





Thin parts are standard

dealing with $\Gamma = \text{discrete} \subset \mathrm{Isom}\,\mathbb{H}^n$

$[I + \varepsilon A, I + \varepsilon B] = I + \mathcal{O}(\varepsilon^2)$

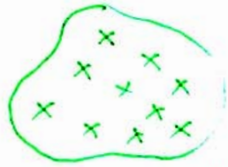
in dim $= 3$ tubes $S^1 \times D^2$

cusps $T^2 \times [0, \infty)$

Gromov Hausdorff Topology

$S = \{$ all complete $\boxed{\text{compact}}$ (initially) metric spaces : uniformly totally bounded $\}$

ε-net

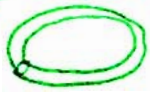


pick a discrete set of pts. s.t. $\forall p$ in space, w/in ε of an element of the ε-net.

uniformly totally bounded / $\forall \varepsilon \; \exists N$ st $\forall X \in S \; \exists$ ε-net for $X \leq N$-pts.

thm/ S is compact, limits are unique

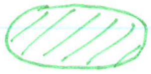
eg/ $(\varepsilon S^1) \times S^1 \xrightarrow[\varepsilon \to 0]{} \bigcirc = S^1$



dimension collapse

Generalize to Non-compact/ pick a base-point, limit depends on the base-point.

Hyperbolic Manifolds in dim ≥ 3

Mostow - Prasad Rigidity/ If M^n hyperbolic, $n \geq 3$, $\mathrm{vol}(M) < \infty$, and M is complete Then hyperbolic structure is unique hyperbolic surface $= F = \mathbb{H}^2/\Gamma$ $\quad \Gamma \circlearrowright \mathbb{H}^2 \sim$ 

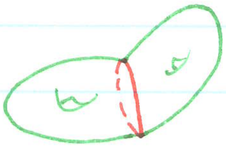
think of $\mathbb{H}^3$ as 

bend by small angle as travel to boundary

$F\times\mathbb{R} \cong \mathbb{H}^3/\Gamma$

this has infinite volume, so counterexample to Vol $<\infty$ case

<u>bending</u>



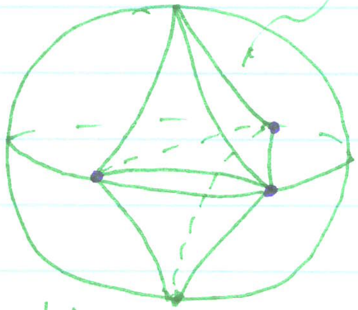
$QF \sim \mathbb{R}^{|\chi|\cdot\underbrace{\dim(SL(2,\mathbb{C}))}_{6}}$

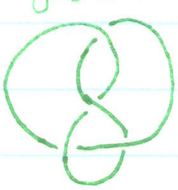
Quasi-Fuchsian

Hyperbolic Dehn Filling

$S^3 \setminus$  $= M_\infty$

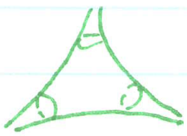
Figure 8-knot



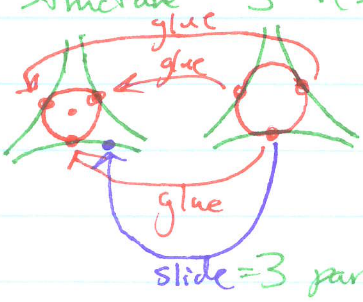


this is the ! complete

<u>Incomplete Structure:</u>



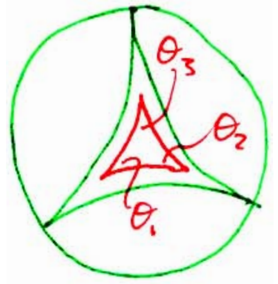
eg) unique complete hyp. structure $S^2\setminus(3\text{ pts.})$



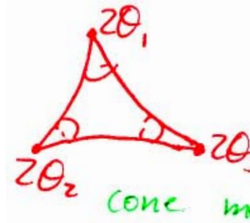
1 way to get complete, otherwise compactify by gluing in S^1's

slide = 3 params for incomplete spaces

eg)



glue like



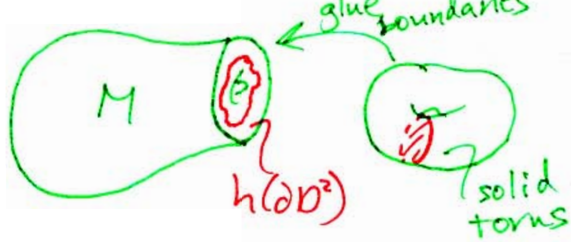
cone metric

$2\theta_i = \frac{2\pi}{n_i} \longleftrightarrow$ discrete group

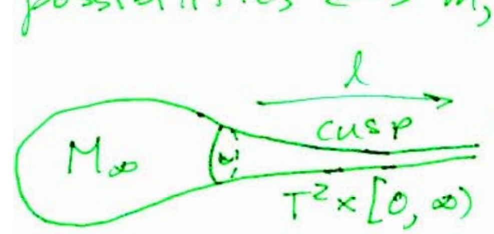
<u>Dehn Filling</u>

M = 3-manifold $\quad \partial M = T$

eg) $M - \mathring{\eta}$(fig 8)



possibilities $\longleftrightarrow m, n \in \mathbb{Z} \Leftrightarrow m/n \in \mathbb{Q} \cup \infty$



shrinks $e^{-\ell}$ $\rightsquigarrow$ incomplete structure

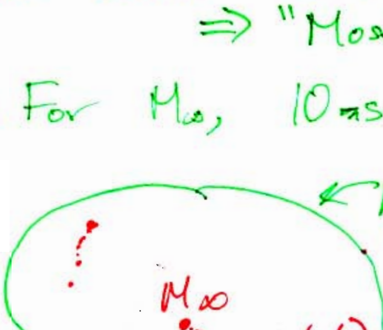
incomplete $\rightsquigarrow$ structure

" Dehn filling

<u>Thurston's Hyperbolic Dehn Surgery Thm</u> All but finitely many Dehn fillings on a cusp of a complete finite volume hyperbolic 3-manifold are hyperbolic and come from deformations

$\Rightarrow$ "Most hyperbolic"

eg) For M_∞, 10 surgeries fail, rest hyperbolic



hyperbolic manifolds w/ singularities

$|p| + |q| \to \infty$

Luckorsh Wallace | Every ~~compact~~ closed orientable 3-manifold is Dehn filling a link

Classification
Produce a complete list w/out repetition (eg prime numbers)

1) List links
2) list Dehn fillings
3) compute hyperbolic structure or show $\nexists$ one
4) check if one is on the list