MATH 145 Supplementary Problems

May 5, 2008

1. Let A be a subset of a topological space X. Prove that A is closed if and only if $\partial A \subset A$, and that A is open if and only if $\partial A \subset X - A$.

2. Prove that $f : (X, \mathcal{T}) \to (Y, \mathcal{T}_i)$ is always continuous, where $\mathcal{T}$ is any topology on X and $\mathcal{T}_i$ is the indiscrete topology on Y.

3. Prove that the unit circle $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ is homeomorphic to ∂A, where A is the square $\{(x, y) \in \mathbb{R}^2 \mid |x| \leq 1, |y| \leq 1\}$.

4. Prove that the functions described in class as the euclidean metric, the square metric, and the component sum metric are indeed metrics on $X \times X$ where $X = \mathbb{R}^2$ and $d_i(x, y) = |x - y|$ for $i = 1, 2$.

5. Prove that the euclidean metric, square metric, and component sum metric satisfy the convergence property described in class (and given below) for $X = \mathbb{R}^n$ and $d_i(x, y) = |x - y|$ for $i = 1, \ldots, n$.

   A sequence $\{\bar{x}^{(j)}\}_{j=1}^{\infty} = \{(x_k^{(j)})\}_{j=1}^{\infty}$ converges to $\bar{y} = (y_1, \ldots, y_n)$ in $X = X_1 \times \cdots \times X_n$ if and only if for each k the sequence of component entries $\{x_k^{(j)}\}_{j=1}^{\infty}$ converges to y_k in X_k.