Midterm 2
Solutions

1. Metric spaces are Hausdorff, and compact subspaces of Hausdorff spaces are closed. $\blacksquare$

2. Let $\mathcal{T}$ be a topology on A and suppose that $p: X \to A$ is continuous.

<u>WTS</u>: $\mathcal{T} \subseteq \mathcal{T}_Q$, where $\mathcal{T}_Q$ is the quotient topology on A induced by p.

Let $U \in \mathcal{T}$. Since p is continuous, we know that $p^{-1}(U)$ is open in X.

Thus, by defn of $\mathcal{T}_Q$, $U \in \mathcal{T}_Q$. $\blacksquare$

3. Suppose, on the contrary, that $A \not\subset B$.

Then $C = A \cap B$ is a proper, nonempty subset of A.

Also notice that C is both open & closed in A since B is both open & closed in X.

$\longrightarrow\!\!\times\!\!\longleftarrow$ A is connected

$\therefore$ $A \subset B$.

4. Let $\mathcal{A}$ be an open cover of $(X, \mathcal{T})$. Since $\mathcal{T} \subseteq \mathcal{T}'$, every element of $\mathcal{T}$ is also open in $(X, \mathcal{T}')$. Thus $\mathcal{A}$ is also an open cover of $(X, \mathcal{T}')$. By compactness of $(X, \mathcal{T}')$ we can reduce $\mathcal{A}$ to a finite subcover of $(X, \mathcal{T}')$. This is also a finite subcover of $(X, \mathcal{T})$.

5. Suppose x, y are two points in A. Then there is a path from x to y since A is path-connected. Similarly, we can find a path from x to y for x, y in B. Now suppose $x \in A$ and $y \in B$. Since $A \cap B \neq \phi$, choose $z \in A \cap B$. There is a path from x to z (since A path-connected) and a path from z to y (since B path-connected). The composition of

these two paths is a path from x to y.

$\square$