MATH 145 Midterm 2
50 points

May 30, 2008

Each problem is worth 10 points.

1. Show that every compact subspace of a metric space is closed.
2. Let $p : X \to A$ be a surjection from the topological space X to the set A. Prove that the quotient topology on A induced by p is the finest topology relative to which p is continuous.
3. A subspace Y of a topological space X is said to be connected if the only two subsets of Y that are both open in Y and closed in Y are Y and $\emptyset$. Let A and B be subspaces of a topological space X. If A is connected, B is open and closed in X, and $A \cap B \neq \emptyset$, prove that $A \subset B$. *Hint: Suppose, on the contrary, that $A \not\subset B$, and show that A must be disconnected.*
4. Let X be a set, and let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on X. Prove that if $\mathcal{T}' \supset \mathcal{T}$ and $(X, \mathcal{T}')$ is compact, then $(X, \mathcal{T})$ is compact.
5. If A and B are path-connected subsets of a topological space X and $A \cap B \neq \emptyset$, then $A \cup B$ is path-connected.