MATH 145 Midterm 1
75 points

May 1, 2008

1. (10 pts.)The "line with two origins" is $X = (\mathbb{R} - \{0\}) \cup \{0^+, 0^-\}$ with the topology on X defined to consist of open sets in $\mathbb{R} - \{0\}$ and of sets of the form $(U - \{0\}) \cup \{0^+\} = U^+$ and $(U - \{0\}) \cup \{0^-\} = U^-$ for $U \subset \mathbb{R}$ open and containing 0. Prove or disprove: X is Hausdorff.

2. (20 pts.)Let A be a subset of a topological space X.
   (a) Prove that $\partial A = \emptyset$ if and only if A both is open and closed.
   (b) Prove that $\bar{A} = A \cup \partial A$.

3. (10 pts.)Let Y be a subspace of a topological space X and let A be a subset of Y. Denote the interior of A in X by $\mathring{A}_X$ and the interior of A in Y by $\mathring{A}_Y$. Prove that $\mathring{A}_X \subset \mathring{A}_Y$.

4. (20 pts.)Let $A \subset X$ and $B \subset Y$. Show that in the space $X \times Y$, $\overline{A \times B} = \bar{A} \times \bar{B}$.

5. (15 pts.)Prove the following statements about continuous functions and discrete and indiscrete topological spaces.
   (a) If X is discrete, then every function f from X to a topological space Y is continuous.
   (b) If X is not discrete, then there is a topological space Y and a function $f : X \to Y$ that is not continuous.
   (c) If Y is not indiscrete, then there is a topological space X and a function $f : X \to Y$ that is not continuous.