Homework 3

Due: Friday, October 17 by 5pm in my mailbox!

October 9, 2008

1. Exercise 2.6: Consider a function $p : \overline{\mathbb{C}} \to \overline{\mathbb{C}}$ of the form $p(z) = \dfrac{az+b}{cz+d}$ where $a, b, c, d \in \mathbb{C}$ and $ad - bc = 0$. Prove that p is not a homeomorphism of $\overline{\mathbb{C}}$.

2. Find the fixed points of a dilation, translation, and the inversion on $\overline{\mathbb{C}}$.

3. If $Tz = \dfrac{az+b}{cz+d}$ is a Möbius transformation of $\overline{\mathbb{C}}$, show that $T(\overline{\mathbb{R}}) = \overline{\mathbb{R}}$ if and only if we can choose a, b, c, d to be real numbers.

4. Evaluate the following cross ratios:
   (a) $[7+i, 1, 0, \infty]$
   (b) $[0, 1, i, -1]$

5. Find a Möbius transformation sending:
   (a) the triple $(1, 0, \infty)$ to the triple $(4, i, -1)$
   (b) the triple $(0, i, -i)$ to the triple $(0, 1, 2)$

6. Find a Möbius transformation that takes:
   (a) the circle $A = \{w \in \mathbb{C} \mid |w+i| = 2\}$ in $\overline{\mathbb{C}}$ to the circle $B = \{w \in \mathbb{C} \mid |w - (1+i)| = 1\}$ in $\overline{\mathbb{C}}$.
   (b) the unit circle S^1 in $\overline{\mathbb{C}}$ to the circle in $\overline{\mathbb{C}}$ defined by the Euclidean line $x + y = 1$.

7. Exercise 2.14: Determine whether $2+3i, -2i, 1-i$, and 4 lie on a circle in $\overline{\mathbb{C}}$.