Homework 1

Due: Friday, October 3

October 1, 2008

1. Find the Cartesian form of these complex numbers:
   (a) $(1+i)(1-i)$
   (b) $(5+10i)(-2+3i)$
   (c) $\dfrac{4+i}{1-2i}$
2. Find the polar form of these complex numbers:
   (a) $-1+i\sqrt{3}$
   (b) $4i$
   (c) $5-i5\sqrt{3}$
3. Prove these properties of conjugate and modulus (z, w are complex numbers):
   (a) $|kz| = |k||z|$ for $k \in \mathbb{R}$
   (b) $|z|^2 = z\overline{z}$
   (c) $|wz| = |w||z|$
   (d) $\overline{zw} = \overline{z}\,\overline{w}$
4. Show that the general complex linear function

   $$Tz = az + b$$

   where a and b are any complex numbers, is the composition of a rotation, followed by a dilation, followed by a translation.
5. The points $1, 1+i, 1-i, 1+2i$ all lie on a line. Plot them and their inversions. The inverted points lie on a circle. What is the equation of this circle?
6. Show that $(\mathbb{C}, \mathcal{E})$ is a geometry, where

   $$\mathcal{E} = \{T : \mathbb{C} \to \mathbb{C} \,|\, Tz = e^{i\theta}z + b, 0 \leq \theta \leq 2\pi, b \in \mathbb{C}\}.$$

7. Exercise 1.1 in book : Express the equations of the Euclidean line $ax + by + c = 0$ and the Euclidean circle $(x-h)^2 + (y-k)^2 = r^2$ in terms of the complex coordinate $z = x + iy$ in $\mathbb{C}$.
8. Exercise 1.2 in book: Let $S^1 = \{z \in \mathbb{C} \,|\, |z| = 1\}$ be the unit circle in $\mathbb{C}$. Let A be a Euclidean circle in $\mathbb{C}$ with Euclidean center $re^{i\theta}$, $r > 1$, and Euclidean radius $s > 0$. Show that A is perpendicular to S^1 if and only if $s = \sqrt{r^2 - 1}$.