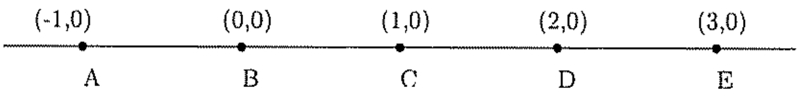
MATH 102A HW 5

8 (2points)
Let l be any line By IA2 there exist at least two distinct points incident with l, call them B, D By BA2 there exist points A, C, and E lying on $\overleftrightarrow{DB}$ such that $A * B * D$, $B * C * D$, and $B * D * E$ By using the same argument as ex 1a of HW4 we get that all 5 points are distinct Thus every line has at least 5 distinct points To prove that every line has an infinite number of points lying on it we will do so by induction We want to prove that any line l has at least n distinct points for any natural number n Base case: By AI2 every line has at least two distinct points, thus our assetion holds for $n = 1, 2$ Induction Hypotesis: Suppose that l has at least n distinct points, call them by their number say $(1), (2), \quad, (n-1), (n)$ Furtheremore by BA3 we can order the points (and after possibly relabeling) to have $(1) * (j) * (n)$ for $j = 2, \quad, n-1$ By BA2 there exist points A, C, E lying on $\overleftrightarrow{(1)(n)}$ such that $A * (1) * (n)$, $(1) * C * (n)$, and $(1) * (n) * E$ If we show that E is distinct from $(1), \quad, (n)$, we would get at least $n+1$ points Since $(1) * (n) * E$, then E is distinct from (1) and (n), if $E = (j)$ for $j = 2, \quad n-1$, then we would bave $(1) * (E) * (n)$ and $(1) * (n) * E$ contraty to BA3 Thus E is distinct from (j) for $j = 1, \quad, n$, and so $l = \overleftrightarrow{(1)(n)}$ has at least n+1 points So by induction every line l has an infinite number of points

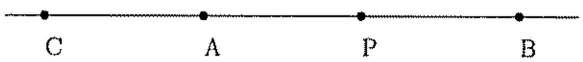
9 (2 points)
Let $P \in \overrightarrow{AB}$ and $P \neq A$ Then there are 3 possibilities: $A * P * B$, $P = B$, or $A * B * P$ If $P = B$ then by defenition of being on the same side, P and B are on the same side of l Suppose P and B are on opposide sides of l Then the segment PB intersects the line l Since the point A is both on l and on the line $\overleftrightarrow{PB}$, we have that this point of intersection must be A Hence we have $P * A * B$, which contradicts (by BA3) the two remaining posibilities Thus P and B are on the same side of l in all cases

15 (2 points)
The hint in the book is very generous To paraphrase, for points, lines and lies on, take the usual interpretation in the usual drawn diagram, but pervert the meaning of betweenness as follows: $A * B * C$ means B is the midpoint of the (usual Euclidean) segment AC All three incidence axioms are obviously true in this interpretation, since the incidence relation was not changed at all Also, the axioms BA1 and BA2 are also true For the sake of an example, take two points in the Cartesian plane, say $B = (0, 0)$ and $D = (2, 0)$ Then the points A, C and E guaranteed to exist by BA2 are the points $A = (-1, 0)$, $C = (1, 0)$ and $E = (3, 0)$ To be sure, B is the midpoint of the segment AC, C is the midpoint of the segment BD and D is the midpoint of the segment CE, so $A * B * C$, $B * C * D$ and $C * D * E$, as required



However, BA3 does not hold in this interpretation Consider the points $B = (0,0)$, $C = (1,0)$ and $E = (3,0)$ in the Cartesian plane None is the midpoint of the others, so none of these three collinear points is between the other two

16 (2 points)
Again as in ex 15 we take for points, lines and incidence as the standard Euclidian interpretation Let A, B and C be 3 distinct colinear points such that $C * B * A$ By BA2 we know that there exist point P such that $A * P * B$ As per hint, A will now be considered between P and B, that is $P * A * B$ But all other betweenes conditions still hold, namely $C * A * P$ To show the failure of the line separation property (prop 3 4), the point P must not lie on either of the rays formed from the relationship $C * A * B$, namely $\overrightarrow{AB}$ or $\overrightarrow{AC}$ If $P \in \overrightarrow{AB}$ then $P = A$, $P = B$, $A * P * B$ or $A * B * P$ Since $P * A * B$ we have that $P \neq B$ and $P \neq A$, and other two conditions contradict BA3 (A is between P and B), thus $P \notin \overrightarrow{AB}$ If $P \in \overrightarrow{AC}$ then $P = A$, $P = C$, $A * P * C$ or $A * C * P$ Since $C * A * P$ we have that $P \neq A$ and $P \neq C$, and other two conditions contradict BA3 (A is between P and C), thus $P \notin \overrightarrow{AC}$



20 (2 points)

1 RAA hypothesis

2 CA1

3 if $G = F$ step 2 contradicts step 1

4 CA3

5 initial hypothesis

6 step 4 and 5

7 step 6 + uniqueness part of C1

8 step 7 contradicts step 3, hence RAA conclusion

24 (2 points)

1 CA1

2 CA6(SAS)

3 definition of congruent triangles

4 CA3 and step 1

5 CA6(SAS) with steps 3, 4

6 definition of congruent triangles

7 CA6(SAS) with steps 1, 6

8 definition of congruent triangles

26 (2 points)

1 CA1

2 CA6(SAS)

3 definition of congruent triangles

4 from step 3, the initial hypothesis that $\angle C \cong \angle F$ and the uniqueness clause in CA4

5 from step 4 + Prop. 2.1 (two distinct nonparallel lines have a unique part in common.)

6 from step 2 and 5