$\underline{\text{Sol}^{\underline{n}}\text{s}}$

1.

$$y = \int_{\cos x}^{5x} \cos(u^2)\,du$$

$$= \int_{1}^{5x} \cos(u^2)\,du + \int_{\cos x}^{1} \cos(u^2)\,du$$

$$= \int_{1}^{5x} \cos(u^2)\,du - \int_{1}^{\cos x} \cos(u^2)\,du$$

FTC $\Longrightarrow$

$$y' = \cos\left((5x)^2\right)\cdot 5 - \cos\left((\cos x)^2\right)\cdot(-\sin x)$$

2. (a)

$$\int e^s \cos(e^s)\,ds \;\overset{u=e^s}{\underset{du=e^s ds}{=\!=}}\; \int \cos u\,du$$

$$= \sin u + C = \sin(e^s) + C$$

(b)

$$\int_0^1 \underbrace{(e-e^x)\,\underbrace{x}_{u}\;}_{dv} ds = x(ex-e^x) - \int (ex-e^x)\,dx$$

$$= x(ex - e^x) - \frac{1}{2}ex^2 + e^x + C.$$

(c) $$\int \frac{\ln^2 x}{x}\,dx \;\underset{du = \frac{1}{x}dx}{\overset{u = \ln x}{=\!=\!=}}\; \int u^2\,du = \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}(\ln x)^3 + C$$

(d) $$\int_0^2 t\sqrt{1+t^2}\,dt \;\underset{du = 2t\,dt}{\overset{u = 1+t^2}{=\!=\!=}}\; \int_1^5 \sqrt{u}\,\frac{1}{2}\,du$$

$$= \frac{1}{2}\cdot\frac{1}{\frac{1}{2}+1}\,u^{\frac{1}{2}+1}\Big|_1^5 = \frac{1}{3}(5\sqrt{5} - 1)$$

(e) $$\int \underbrace{\sin 3x}_{u}\,\underbrace{e^{2x}dx}_{dv} = \sin 3x\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,3\cos 3x\,dx$$

$$= \frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\int \underbrace{\cos 3x}_{u}\,\underbrace{e^{2x}dx}_{dv}$$

$$= \frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\left[\cos 3x\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}(-3\sin 3x)\,dx\right]$$

$$= \frac{1}{2}e^{2x}\sin 3x - \frac{3}{4}e^{2x}\cos 3x - \frac{9}{4}\int \sin 3x\,e^{2x}\,dx$$

$$\Rightarrow \left(1 + \frac{9}{4}\right)\int \sin 3x\,e^{2x}\,dx = \frac{1}{2}e^{2x}\sin 3x - \frac{3}{4}e^{2x}\cos 3x$$

Hence

$$\int \sin 3x\, e^{2x}\, dx = \frac{2}{13} e^{2x} \sin 3x - \frac{3}{13} e^{2x} \cos 3x + C$$

(f). $$\int \sin^2 x \cos^3 x\, dx = \int \sin^2 x \cos^2 x \underline{\cos x\, dx}.$$

$\underline{\underline{u = \sin x}}$
$du = \cos x\, dx$
$$\int u^2 (1-u^2)\, du = \frac{1}{3} u^3 - \frac{1}{5} u^5 + C$$

$$= \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$$

(g) $$\int_{-\infty}^{+\infty} \frac{x}{\sqrt[3]{x^2+1}}\, dx = \int_0^{+\infty} \frac{x}{\sqrt[3]{x^2+1}}\, dx$$

$$+ \int_{-\infty}^{0} \frac{x}{\sqrt[3]{x^2+1}}\, dx$$

$$= \lim_{t\to\infty} \int_0^t \frac{x}{\sqrt[3]{x^2+1}}\, dx + \lim_{s\to-\infty} \int_s^0 \frac{x}{\sqrt[3]{x^2+1}}\, dx$$

$\underline{\underline{u = x^2+1}}$
$du = 2x\,dx$
$$\lim_{t\to\infty} \int_1^{t^2+1} \frac{\frac{1}{2}du}{\sqrt[3]{u}} + \lim_{s\to-\infty} \int_{s^2+1}^{1} \frac{\frac{1}{2}du}{\sqrt[3]{u}}$$

$$= \lim_{t\to\infty} \frac{3}{4} u^{2/3} \Big|_1^{t^2+1} + \lim_{s\to-\infty} \frac{3}{4} u^{2/3} \Big|_{s^2+1}^{1}$$

$$= \lim_{t\to\infty} \left[\frac{3}{4}(t^2+1)^{2/3} - \frac{3}{4} 2^{2/3} \right] + \lim_{s\to-\infty} (\quad)$$

$$= \infty + \lim_{s \to -\infty} (\quad)$$

$$\Longrightarrow \int_{-\infty}^{\infty} \frac{x}{\sqrt[3]{x^2+1}}\,dx \quad \text{DNE/divergent}$$

(h) $\dfrac{x^2}{x^2+5x-6}$

$$\begin{array}{r} 1 \\ x^2+5x-6 \overline{\big)\, x^2 \qquad\qquad} \\ -)\ \ x^2+5x-6 \\ \hline -5x+6 \end{array}$$

$$= 1 + \frac{-5x+6}{x^2+5x-6}$$

$$x^2+5x-6 = (x+6)(x-1) \Longrightarrow$$

$$\frac{-5x+6}{x^2+5x-6} = \frac{A}{x+6} + \frac{B}{x-1}$$

$$\Longrightarrow \quad -5x+6 = A(x-1) + B(x+6)$$

$$x=-6: \quad 36 = -7A \Rightarrow A = -\frac{36}{7}$$

$$x=1: \quad 1 = B\cdot 7 \Rightarrow B = \frac{1}{7}$$

$$\Longrightarrow \int \frac{x^2}{x^2+5x-6}\,dx = \int \left(1 + \frac{-36/7}{x+6} + \frac{1/7}{x-1}\right)dx$$

$$= x + \frac{1}{7}\ln|x-1| - \frac{36}{7}\ln|x+6| + C.$$

(i). $\displaystyle\int_1^e \overbrace{(e-x)\underbrace{\ln x}_{u}\,dx}^{dv} = -\cancel{(\ln x)\frac{1}{2}(e-x)^2}\Big|_1^e \to 0$

$$+ \int_1^e \frac{1}{2}(e-x)^2 \cdot \frac{1}{x}\,dx = \frac{1}{2}\int_1^e \left[\frac{e^2}{x} - 2e + x\right]dx$$

$$= \frac{1}{2}\left[e^2 \ln|x| - 2ex + \frac{1}{2}x^2 \right]\Bigg|_1^e$$

$$= \frac{1}{2}\left[e^2 - 2e^2 + \frac{1}{2}e^2 - 0 + 2e - \frac{1}{2} \right]$$

$$= -\frac{1}{4}e^2 + e - \frac{1}{4}$$

(j) $\dfrac{1}{x^2-6x+5} = \dfrac{1}{(x-5)(x-1)}$ has an ∞

discont. at $x = 1$

$$\Rightarrow \int_0^1 \frac{1}{x^2-6x+5}\,dx = \lim_{t\to 1^-} \int_0^t \frac{1}{(x-5)(x-1)}\,dx$$

$$\frac{1}{(x-5)(x-1)} = \frac{A}{x-5} + \frac{B}{x-1}$$

$$\Rightarrow 1 = A(x-1) + B(x-5)$$

$$x=5 \Rightarrow 1 = 4A, \quad A = \frac{1}{4}$$

$$x=1 \Rightarrow 1 = -4B, \quad B = -\frac{1}{4}$$

$$\Rightarrow \int_0^1 \frac{1}{x^2-6x+5}\,dx = \lim_{t\to 1^-} \int_0^t \left[\frac{1}{4}\frac{1}{x-5} - \frac{1}{4}\cdot\frac{1}{x-1} \right] dx$$

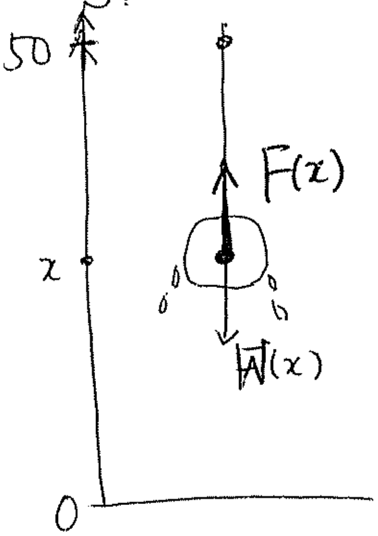
$$= \lim_{t\to 1^-} \left(\frac{1}{4}\ln|x-5| - \frac{1}{4}\ln|x-1| \right)\Bigg|_0^t$$

$$= \lim_{t\to 1^-} \left(\underbrace{\frac{1}{4}\ln|t-5|}_{\to \frac{1}{4}\ln 4} - \underbrace{\frac{1}{4}\ln|t-1|}_{\to -\infty} - \frac{1}{4}\ln 5 + \frac{1}{4}\cdot 0 \right)$$

$= +\infty$

$$\Rightarrow \int_0^1 \frac{1}{x^2-6x+5}\,dx \quad \text{is divergent}$$

3.



Let x = height in ft

$F(x)$ = force acting when height is x

= weight of ice " " " "

Then

$$W = \int_0^{50} F(x)\,dx$$

Now the ice is lifted at a rate of 1 ft/min and is melting at a rate of 2 lb/min

$\Rightarrow$ at x ft, weight of ice

$$= \underbrace{1000}_{\text{initial weight}} - \underbrace{\frac{x}{1}}_{\substack{\text{time taken} \\ \text{to go to } x \text{ ft.}}} \times \underbrace{2}_{\text{rate of melting}}$$

$$= 1000 - 2x$$

$$\Rightarrow \quad W = \int_0^{50} (1000 - 2x)\,dx = 47{,}500 \text{ ft-lb}$$

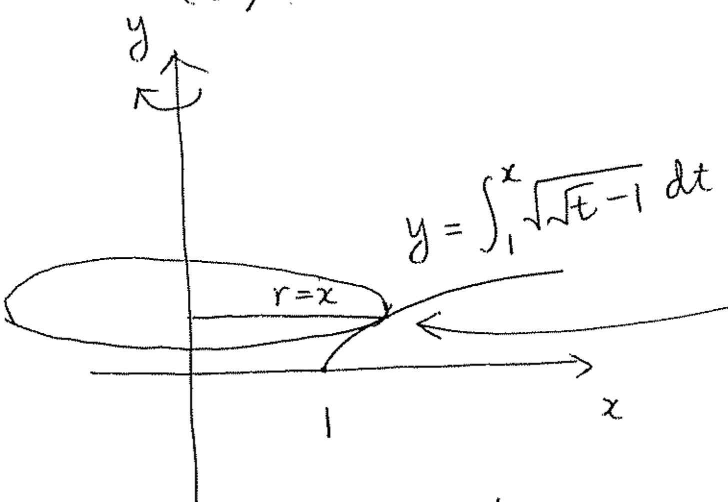
4. (a) FTC $\Longrightarrow$

$$y' = \sqrt{\sqrt{x} - 1}$$

$$\Longrightarrow \quad L = \int_1^{16} \sqrt{1+(y')^2}\, dx = \int_1^{16} x^{1/4}\, dx = \frac{4}{5} x^{5/4} \Big|_1^{16}$$

$$= \frac{124}{5} \text{ meter (if the unit is meter.)}$$

(b)



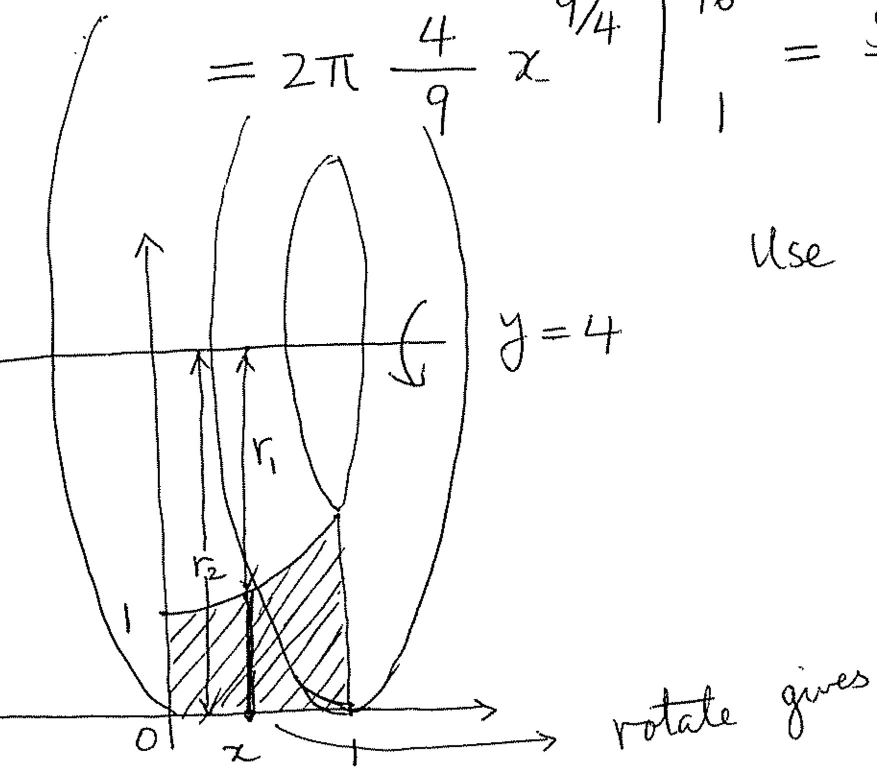
$$S = \int_1^{16} 2\pi x \, ds$$

$$= \int_1^{16} 2\pi x \sqrt{1+(y')^2}\, dx$$

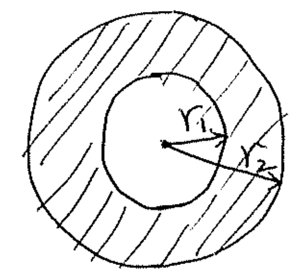
$$= \int_1^{16} 2\pi\, x \cdot x^{1/4}\, dx = 2\pi \int_1^{16} x^{5/4}\, dx$$

$$= 2\pi \frac{4}{9} x^{9/4} \Big|_1^{16} = \frac{504\pi}{9} = 56\pi$$

5.



Use disc/washer method



$$r_1 = 4 - y$$
$$= 4 - e^{x/3}$$
$$r_2 = 4.$$

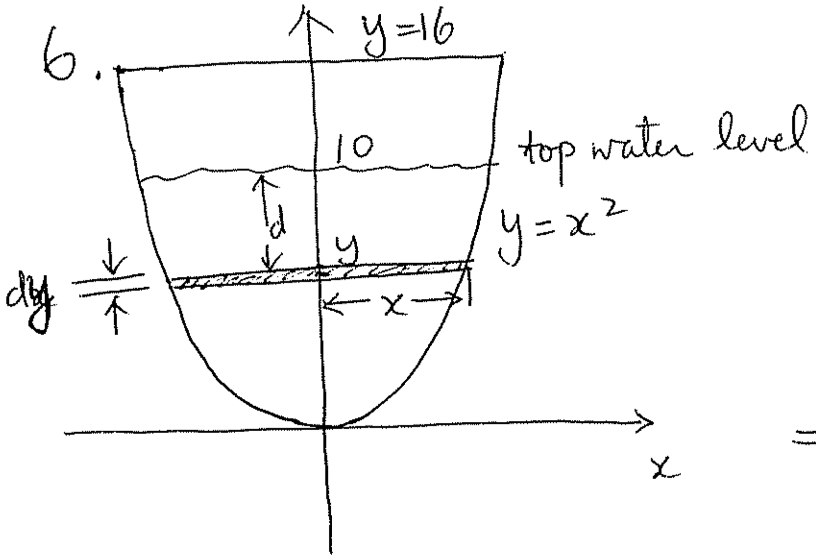
$$\Rightarrow \quad V = \int_0^1 \pi \left[4^2 - (4 - e^{x/3})^2 \right] dx$$

$$= \pi \int_0^1 \left(8 e^{x/3} - e^{2x/3} \right) dx$$

$$= \pi \left(\frac{8}{1/3} e^{x/3} - \frac{1}{2/3} e^{2x/3} \right) \Bigg|_0^1$$

$$= \pi \left(24 e^{1/3} - \frac{3}{2} e^{2/3} - 24 + \frac{2}{3} \right)$$

6.



Let dF = hydrostatic force acting on the horizontal strip located at $\underline{\underline{y}}$ of thickness $\underline{\underline{dy}}$.

$$\Rightarrow \quad dF = \underset{\substack{\uparrow \\ \text{pressure}}}{P} \cdot \underset{\substack{\uparrow \\ \text{area of strip}}}{\underline{dA}}$$

Now

$$P = \rho g d = 1000 \times 9.8 \times (10 - y)$$

$$dA = 2x \, dy \xrightarrow[\Rightarrow x = \sqrt{y}]{y = x^2} 2\sqrt{y} \, dy$$

$$\Rightarrow \quad dF = 9800 (10 - y) \cdot 2\sqrt{y} \, dy$$

$$\Rightarrow \quad F = \int_0^{10} dF = 19600 \int_0^{10} (10 - y) \sqrt{y} \, dy$$

$$= 19600 \left(\frac{10}{\frac{1}{2}+1} y^{1/2+1} - \frac{1}{\frac{3}{2}+1} y^{3/2+1} \right) \Bigg|_0^{10}$$

$$= \frac{1568}{3}\sqrt{10} \quad J.$$