240B HW 3 Winter 2013 Due: Friday, 02/08.

1. do Carmo p58: 7.

2. do Carmo p58: 8.

3. Let G be a Lie group. Show that there is a unique (affine) connection ∇ such that

$$\nabla_X Y = 0,$$

for all left invariant vector fields X, Y. Show also that, in the case when G is connected, this connection is torsion free if and only if G is abelian. (**Hint:** you can use the fact that a connected Lie G is abelian if and only if [X, Y] = 0 for all left invariant vector fields X, Y.)

4. Check "by hand" that the geodesic field defined in class is well-defined.