240B HW 2 Winter 2013

Due: Friday, 1/25.

1. do Carmo, p46, 7.

2. do Carmo, p56, 2 (assume that the connection is affine, instead of Riemannian).

3. do Carmo, p57, 3 (again, deal with affine connections only).

4. Let (M^n, g) be a Riemannian manifold and f a smooth function on M. Define the gradient ∇f of f to be the vector field such that

$$\langle \nabla f,\ X \rangle = Xf (= df(X)),$$

for any vector field X on M. Let $x = (x_1, \cdots, x_n)$ be a local coordinate system on M and $g = g_{ij} dx_i dx_j$. Define g^{ij} to be the entries of the inverse of the matrix (g_{ij}):

$$(g^{ij}) = (g_{ij})^{-1}.$$

Show that

$$\nabla f = (g^{ij} \frac{\partial f}{\partial x_i}) \frac{\partial}{\partial x_j}.$$

Hence it agrees with the usual gradient in the Euclidean space.