

## Midterm Check List, Math 3C, Crandall, Winter 2003: Version 1, 2/5/03

\*\*BRING PHOTO ID\*\*

\*\*EXAM IS CLOSED BOOK, NO CALCULATORS\*\*

The purpose of this handout is to provide you with information about the sort of problems which could occur on the midterm exam. It should also help you figure out how to prepare for the exam. For example, for the “action” problems in Section 1, the first thing is to make sure you understand what is being said and then to refresh your memory of how to do the actions described. The webwork sets provide review and practice. By the time you have done this, your vocabulary should be pretty much up to date, and Section 2 “talk” type problems, will not take much further time to prepare for. Maybe you will want to make a list of terms and your definitions of them for a bit of practice in advance of the exam. Section 3 “explain” type problems will be about things which are important in your instructor’s opinion and which were repeated more than once. Your answers to them indicate how much effort you put into understanding things that you should try to understand. They go beyond “see this, do that”. Section 4 “true-false” type problems, and Section 5 “example” type problems are not easy to prepare for at the last minute. Your answers to them reflect the attention you have paid to the material in lecture, in the book, and the extent to which you have tried to understand everything. Section 6 problems are self-explanatory.

DISCLAIMER: Probably only some of the problem types mentioned here will appear on the midterm. The midterm has not been created yet, so there is no way to know. :) However, preparation for doing any of them will help with others of them.

CLAIMER: However, Section 1 problems will certainly be the single most important category.

### 1. “ACTION” TYPE PROBLEMS

This list is pretty complete. It tells you things you should be able to do.

- compute and evaluate partial derivatives of given functions of two variables
- compute the equations of tangent planes to graphs of functions of two variables
- draw or identify sections of functions of two variables
- draw or identify level curves of functions of two variables
- sketch or identify slope fields of first order ordinary differential equations (“fodes” here after)
- sketch or identify isoclines for fodes
- given a slope field of a fode, sketch what a solution might look like
- identify equilibrium solutions of fodes
- determine the basins of attraction of equilibrium solutions of fodes from their slope fields
- identify separable fodes
- implicitly solve separable fodes (this means to find relations like  $G(y) = F(t) + c$  satisfied by nonconstant solutions in particular cases)
- implicitly solve ivps for separable fodes
- explicitly solve separable fodes and ivps for them when possible
- given an ivp for a fode, implement Euler’s method BY HAND for a couple steps (maybe backwards! - as in  $y' = ty^2, y(1) = 1, h = .5$ , approximate  $y(0)$ )
- identify linear fodes
- identify homogeneous linear fodes
- identify inhomogeneous linear fodes
- given a particular solution of a linear fode, find the general solution
- find general solutions of linear fodes even if a particular solution is not given

- find an integrating factor for a linear fode
- solve ivps for linear fodes by the method of your choice
- given the general formula for the solution of the general ivp for a linear fode, plug a particular fode and ivp into to it

## 2. “TALK” TYPE PROBLEMS

These are just examples here. You might be asked to define any of the terms appearing above. We want you to learn the vocabulary of the course.

- Define what it means to be an “equilibrium solution” of a fode.
- Define “isocline”.
- Define “integrating factor”.
- Define “round off error” and “discretization error”.
- State the chain rule.
- State the Fundamental Theorem of Calculus.

## 3. “EXPLAIN” TYPE PROBLEMS

We use the friendlier term “explain” rather than “prove”, in a couple of these, but we want a good, convincing, mathematical explanation. ”Explain” does not mean to use words only, you must write equations and justify them, with words, as appropriate. These are just examples.

- If  $y$  is a nonconstant solution of  $y' = g(y)f(t)$ ,  $G' = 1/g$  and  $F' = f$ , explain why then  $G(y) - F(t)$  is constant.
- Explain exactly how to implement Euler’s method for the ivp  $y' = f(t, y)$ ,  $y(t_0) = y_0$ .
- Let  $Ly = y' + p(t)y$ ,  $Ly_1 = t$  and  $Ly_2 = t$ . Explain why  $L(y_1 - y_2) = 0$ .

## 4. TRUE FALSE PROBLEMS

Again, these are just examples.

- An isocline for  $y' = f(t, y)$  is the same thing as a level curve for  $f$ .
- An isocline for  $y' = f(t, y)$  is the same thing as a  $t$ -section of  $f$ .
- $y' = ty^2$  is a linear fode
- $y' = t^2y + 1$  is a linear fode
- $y' = t^2y + 1$  is a homogeneous linear fode
- No separable fode is a linear fode.
- Since there is no formula for an antiderivative of  $\sin(e^t)$ ,  $\int_0^1 \sin(e^s) ds$  does not exist.
- Since there is no formula for an antiderivative of  $\sin(e^t)$ ,  $\int \sin(e^t) dt$  does not exist.
- For all numbers  $a$  &  $b$ ,  $e^{a+b} = e^a e^b$ .
- A ball is  $x$  meters from the top of the Eiffel tower at  $t$  minutes after midnight tonight. At midnight it is at the top of the tower. Its velocity is  $e^t$ . Then  $x = te^t$ .

## 5. “EXAMPLE” TYPE PROBLEMS

Again, these are just examples.

- Give an explicit example of a linear inhomogeneous fode.
- Give an explicit example of an ivp for a fode whose solution “blows up”.
- Give an example of a fode which is not separable and not linear.

## 6. MORE CHAIN RULE AND THE FUNDAMENTAL THEOREM OF CALCULUS

We might ask you to show your understanding of the chain rule and the Fundamental Theorem of Calculus by questions like these.

- Let  $f$ ,  $F$  and  $g$  be functions and  $g(x) = F(f(x))$  for all  $x$ . Given

$x$	$F(x)$	$F'(x)$	$f(x)$	$f'(x)$
1	6	-11	$\pi$	-7
$\pi$	15	-23	31	13
-7	17	-11	12	57
6	$e$	-37	12.8	101

What  $g'(1)$ ?

- Suppose now that  $g(x) = \int_{-\pi}^x F(\tau) d\tau + 16$  for all  $x$ . What is  $g(-\pi)$ ? Using the table above, evaluate  $g'(-7)$ .