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Advances in numerical Micromagnetics: Spin-polarized transport

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Abstract

We consider a model for spin-polarized transport that couples the Landau-Lifshitz-Gilbert equation describing the dynamics of the magnetization with a diffusion equation describing the spin accumulation. The diffusion equation has bounded, discontinuous coefficients. We describe an unconditionally stable method for the coupled system based on a fractional step for the spin equation, and the Gauss-Seidel Projection Method for micromagnetics simulations.

Key words: *Landau-Lifshitz, Micromagnetics, spin-polarized transport*
AMS subject classifications: *65M06; 65Z05; 35Q60.*

1 Introduction

For decades, ferromagnetic materials have been used for the storage of information in hard drives and magnetic tapes. The discovery of Giant Magneto-Resistance has opened new possibilities for the design of non-volatile magnetic storage devices, such as magnetic memories, or MRAMs. In an MRAM, layered structures consisting of two or more ferromagnetic films separated by a layer of nonmagnetic material are connected in an electronic circuit forming a grid (see fig. 1). Each multilayer represents one bit of information, and it has two possible states, identified as 0 and 1.

Traditionally, the magnetization is switched between state 0 and state 1 using a magnetic field generated by flowing currents along the data lines (see fig. 1). The magnetic field generated in this way has long range, and therefore it is difficult to achieve large bit densities. A new mechanism for magnetization reversal based on spin polarized transport was predicted by Slonczewski [1] and Berger [2], and has been the the object of much research in the physics community in the past few years (see [3] for a review). In this new approach, a current flows perpendicular to the multilayer. The electron spin is polarized in layer FM1. When the electrons reach layer FM2, the spin exerts an additional torque on the underlying magnetization.

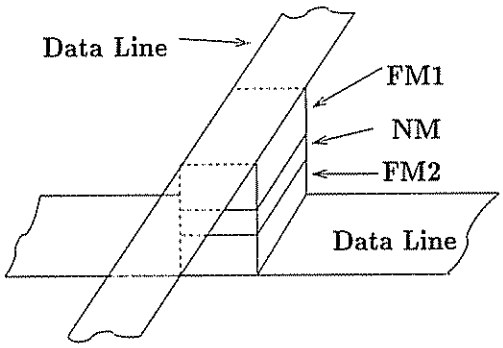

Figure 1: Schematic of a multilayer composed of two ferromagnetic layers (FM1 and FM2) separated by a non-magnetic layer (NM). Currents flow both along the data lines and perpendicular to the layers in the reading/writing process.

In the model introduced by Slonczewski [1], both interfacial effects and spin diffusion are neglected. These effects have been found to be important in magneto resistance experiments with a current perpendicular to the multilayer planes [4]. Recently a new model for the relaxation of the coupled system spin-magnetization has appeared in the literature [5]. This model includes spatial variations in both spin and magnetization, and does not assume a priori that the magnetization is pinned in one of the layers. In [6] and [7], García-Cervera and Wang have generalized the model described in [5] to three dimensions.

In this article we consider a magnetic multilayer consisting of two ferromagnetic films FM1 and FM2, of thickness d_1 and d_2, respectively, separated by a non-magnetic interlayer NM of thickness d_0 (see Fig. 1). The multilayer occupies the volume $\Omega = \text{FM1} \cup \text{NM} \cup \text{FM2}$. The spin accumulation $\mathbf{s}$ is defined on Ω and the magnetization $\mathbf{M}$ is defined on the two magnetic layers $\Sigma = \text{FM1} \cup \text{FM2}$. We will assume that the boundaries of Ω and Σ are smooth.

The dynamics of the coupled system for the spin and the magnetization are described by the system of equations

$$\frac{\partial \mathbf{s}}{\partial t} = -\text{div}\, \mathbf{J}_\mathbf{s} - \frac{2D_0(\mathbf{x})}{\lambda_{sf}^2}\mathbf{s} - \frac{2D_0(\mathbf{x})}{M_s \lambda_J^2}\mathbf{s} \times \mathbf{M}, \quad \mathbf{x} \in \Omega, \tag{1}$$

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times (\mathcal{H} + J\mathbf{s}) + \frac{\alpha}{M_s}\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}, \quad \mathbf{x} \in \Sigma. \tag{2}$$

$\mathbf{J}_\mathbf{s}$ is the spin current, $D_0(\mathbf{x})$ is the diffusion coefficient, λ_{sf} is the spin-flip characteristic length, λ_J is the interaction length scale [5], and $|\mathbf{M}| = M_s$ is constant below the Courie temperature. The spin current is

$$\mathbf{J}_\mathbf{s} = \frac{\beta \mu_B}{e M_s}\mathbf{M} \otimes \mathbf{J}_e - 2D_0(\mathbf{x})\left[\nabla \mathbf{s} - \frac{\beta\beta'}{M_s^2}\mathbf{M} \otimes (\nabla \mathbf{s} \cdot \mathbf{M})\right], \tag{3}$$

where $\mathbf{J}_e$ is the applied electric current, $\mu_B = 9.2741 \times 10^{-24} Am^2$ is the Bohr magneton, $e = -1.602 \times 10^{-19} As$ is the charge of the electron, and $0 < \beta, \beta' < 1$ are the spin-polarization parameters of the two layers,

Equation (2) is the Landau-Lifshitz-Gilbert equation [8, 9], and describes the relaxation process of the magnetization inside Σ. The first term on the right hand side in (2) is the gyromagnetic term, and γ is the gyromagnetic ratio. The second term on the right hand side is the Gilbert damping, and $\alpha > 0$ is the damping parameter. J is the strength of the interaction between the spin and the magnetization. The gyromagnetic term describes the precession of $\mathbf{M}$ around the $\mathcal{H} + J\mathbf{s}$, whereas the damping term accounts for dissipation mechanisms in the system. The field $\mathcal{H}$ is

$$\mathcal{H} = -\frac{K_u}{M_s^2}\left(M_2\mathbf{e}_2 + M_3\mathbf{e}_3\right) + \frac{C_{ex}}{M_s^2}\Delta\mathbf{M} + \mu_0\mathbf{H}_e - \mu_0\nabla U, \tag{4}$$

where we have used $\mathbf{e}_2 = (0,1,0)$ and $\mathbf{e}_3 = (0,0,1)$.

The first two terms in (4) are the anisotropy and exchange fields, respectively, with K_u and C_{ex} being material constants. $\mathbf{H}_e$ is an externally applied field, and μ_0 is the permeability of vacuum ($\mu_0 = 4\pi \times 10^{-7} N/A^2$). Finally, $\mathbf{H}_s = -\nabla U$ is the stray field. The magnetostatic potential, U, satisfies equation

$$\begin{aligned} \Delta U &= \operatorname{div}\mathbf{M}, \quad \mathbf{x} \in \Sigma, \\ \Delta U &= 0, \quad \mathbf{x} \in \mathbb{R} \setminus \overline{\Sigma}, \end{aligned} \tag{5}$$

with jump boundary conditions at the material-vacuum interface:

$$[U]_{\partial\Sigma} = 0; \quad \left[\frac{\partial U}{\partial \nu}\right]_{\partial\Sigma} = -\mathbf{M}\cdot\nu, \tag{6}$$

where ν is the outward unit normal on $\partial\Sigma$.

Equations (1)-(2) must be solved with Neumann boundary conditions

$$\frac{\partial \mathbf{s}}{\partial \mathbf{n}}|_{\partial\Omega} = 0; \quad \frac{\partial \mathbf{M}}{\partial \nu}|_{\partial\Sigma} = 0, \tag{7}$$

where $\mathbf{n}$ is the outward unit normal on $\partial\Omega$.

2 Numerical Results

Explicit time-stepping schemes for the numerical solution of (1)-(2) suffer from a very strict stability constraint [10]. Here we consider a splitting algorithm for the numerical solution of (1)-(2). Given $\mathbf{m}^n, \mathbf{s}^n$, we solve analytically the following system of linear ordinary differential equations:

$$\begin{aligned} \frac{\partial \widetilde{\mathbf{s}}}{\partial t} &= -2D_0(\mathbf{x})\frac{\widetilde{\mathbf{s}}}{\lambda_{sf}^2} - 2D_0(\mathbf{x})\frac{\widetilde{\mathbf{s}} \times \mathbf{m}^n}{\lambda_J^2}, \\ \widetilde{\mathbf{s}}(t_n, \mathbf{x}) &= \mathbf{s}^n(\mathbf{x}). \end{aligned} \tag{8}$$

We consider the intermediate value $\mathbf{s}^*(\mathbf{x}) = \widetilde{\mathbf{s}}(t_{n+1}, \mathbf{x})$. Now we solve

$$\frac{\mathbf{s}^{n+1} - \mathbf{s}^*}{\Delta t} = \nabla\cdot\left(2D_0(\mathbf{x})\left[\nabla\mathbf{s}^{n+1} - \frac{\beta\beta'}{M_s^2}\mathbf{M}^n \otimes (\nabla\mathbf{s}^n\cdot\mathbf{M}^n)\right] - \frac{\beta\mu_B}{eM_s}\mathbf{M}^n \otimes \mathbf{J}_e^n\right).$$

The spatial discretization employed is second order accurate. Finally, the LLG equation (2) is advanced in time using the *Gauss-Seidel Projection Method* (GSPM). The GSPM is an unconditionally stable method for the LLG, whose numerical complexity is comparable to that of solving the linear heat equation using Backward Euler [10, 11]. Further details may be found in [7].

In figure 2 we demonstrate the effect of the spin current in a double layer. The dimensions of the multilayer are 128 $nm \times 64$ $nm \times \{100+20+60\}$ nm. We used the parameters of the Permalloy ($C_{ex} = 1.3 \times 10^{-11} J/m$, $K_u = 10^2 J/m^3$, $M_s = 8 \times 10^5 A/m$, $\gamma = 1.76 \times 10^{11} (Ts)^{-1}$). The current used was $J_e = (0, 0, -10^{11}) A/m^2$. The coupling parameter was $J = 3.125 \times 10^{-4} N/A^2$. The polarization parameter for FM1 was $\beta = 0.9$, and for FM2 was $\beta' = 0.8$. The diffusion constant was set to be $D_0 = 10^{-3} m^2/s$ inside the ferromagnetic layers, and $D_0 = 5 \times 10^{-3} m^2/s$ outside. In the hysteresis loop we plot the average magnetization at steady state for a descending sequence of external fields. The sample is initially saturated by applying a magnetic field $\mathbf{H}_e = 0.06$ mT.

It is seen that when only the magnetic fields are considered, a field of strength 0.06 mT is not enough to reverse the magnetization. However, when spin currents are present, the magnetization reverses with a field of approximately 0.025 mT.

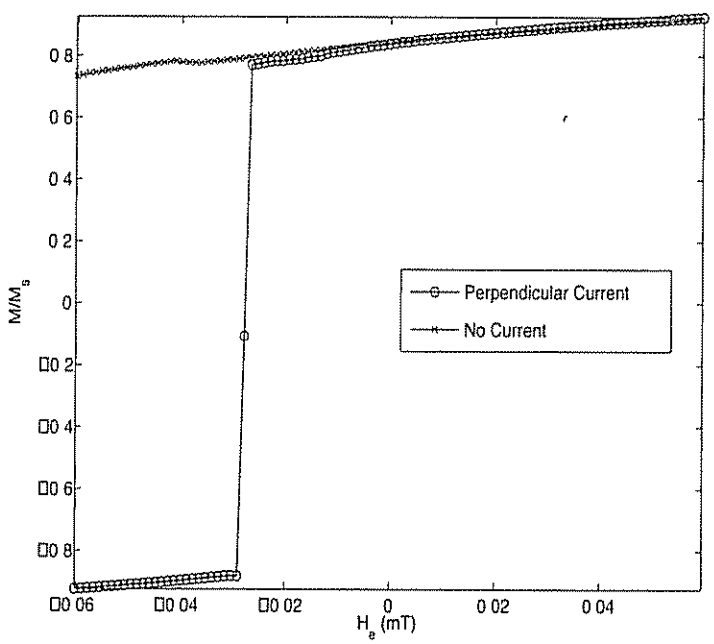

Figure 2: Hysteresis loop in a double layer. Letting a current flow perpendicular to the layers can reduce considerably the magnetic field necessary for magnetization reversal.

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References

[1] J.C. Slonczewski. Current-driven excitation of magnetic multilayers. *J. Magn. Magn. Mat.*, 159:L1–L7, 1996.

[2] L. Berger. Emission of spin waves by a magnetic multilayer traversed by a current. *Phys. Rev. B*, 54:9353, 1996.

[3] M.D. Stiles and J. Miltat. Spin transfer torque and dynamics. *preprint*, 2005.

[4] T. Valet and A. Fert. Theory of the perpendicular magnetoresistance in magnetic multilayers. *Phys. Rev. B*, 48:7099, 1993.

[5] A. Shpiro, P.M. Levy, and S. Zhang. Self-consistent treatment of nonequilibrium spin torques in magnetic multilayers. *Phys. Rev. B*, 67:104430, 2003.

[6] C.J. García-Cervera and X.P. Wang. Spin-Polarized transport: Existence of weak solutions. *Submitted to J. Disc. Cont. Dyn. Sys., Series B*, 2005.

[7] C.J. García-Cervera and X.P. Wang. Numerical simulations of a spin transfer model. *in preparation*, 2005.

[8] L. Landau and E. Lifshitz. On the theory of the dispersion of magnetic permeability in ferromagnetic bodies. *Physikalische Zeitschrift der Sowjetunion*, 8:153–169, 1935.

[9] T.L. Gilbert. *Phys. Rev.*, 100:1243, 1955. [Abstract only; full report, Armor Research Foundation Project No. A059, Supplementary Report, May 1, 1956 (unpublished)].

[10] X.-P. Wang, C.J. García-Cervera, and W. E. A Gauss-Seidel Projection Method for the Landau-Lifshitz equation. *J. Comp. Phys.*, 171:357–372, 2001.

[11] C.J. García-Cervera and W. E. Improved Gauss-Seidel Projection Method for Micromagnetics Simulations. *IEEE Trans. Mag.*, 39(3):1766–1770, 2003.