Math 206D, Fall 2006

The Finite Element Method

Tuesday & Thursday, 2:00-3:15pm, South Hall 6635

Instructor: Carlos J. García-Cervera.

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Office Hours: Tuesday & Thursday, 12:30-2:00pm.

Textbooks: The main reference for the course will be the book by Claes Johnson. However I will extract some material from the other books listed below:

- *Numerical Solution of Partial Differential Equations by the Finite Element Method*, by Claes Johnson.
- *Computational Differential Equations*, by by K. Eriksson, D. Estep, P. Hansbo, and C. Johnson.
- *Finite elements for electrical engineers*, by Peter P. Silvester and Ronald L. Ferrari.
- *The Mathematical Theory of Finite Element Methods*, by Susanne C. Brenner and L. Ridgway Scott.
- *A Multigrid Tutorial*, by William L. Briggs, Van Emden Henson, and Steve F. McCormick.
- *Multilevel Adaptive Methods for Partial Differential Equations*, by Steve F. McCormick.
- *Numerical Solution of Partial Differential Equations*, by K.W. Morton and D.F. Mayers.
- *Finite Element Method: Its basis and Fundamentals*, by O.C. Zienkiewicz, R.L. Taylor, and J.Z. Zhu.

You can also find some Finite Element Method resources and code at the URL `http://www.engr.usask.ca/~macphed/finite/fe_resources`.

For a review of Advanced Calculus, you may want to check out the following:

- *Vector calculus*, by Jerrold E. Marsden and Anthony J. Tromba.

Course description: This is a graduate course in Numerical Analysis, with an emphasis in the solution of Partial Differential Equations by the Finite Element Method (FEM). In this course we will study the FEM for elliptic, hyperbolic, and parabolic equations, and we will also cover the boundary element method. Although previous knowledge in Numerical Analysis is recommended, it is not a requirement for this course. Therefore I will cover some basic material, such as function approximation, and the solution of linear systems of equations. During the course I will illustrate the methods studied with applications from Fluid Dynamics, and Electromagnetism. Although the emphasis will be in applications, the course will have a strong theoretical component.

Prerequisites: Knowledge of a computer language suitable for numerical computing: FORTRAN, C, C++, or Matlab. Previous knowledge in Numerical Analysis is recommended, although not necessary.

Assignments and grading: Homework will be assigned every two weeks. It will be handed out on Fridays, and will be collected at the beginning of the class on the Friday of the second following week. Late homeworks will not be accepted. The homework will generally consist of some theoretical questions, and some computational assignments. You will be required to write a program to solve certain problems. The program must be given to me as part of the assignment, together with the output of the program, in the format indicated in the assignment, and an interpretation of the results whenever necessary. You can write the programs either in FORTRAN, C, C++, or Matlab.

In addition to the homework, there will be a final project. Your final grade for the course will be decided according to the following formula:

Final Grade = 60% Homework + 40% Final

Syllabus: During this course we will try to follow the following schedule. However, much like everything said earlier, this is subject to change.

Week 1: Introduction to the FEM for elliptic problems.

1. Variational formulation of a one-dimensional model problem.
2. FEM for the model problem with piecewise linear functions.
3. An error estimate for FEM for the model problem.
4. FEM for the Poisson equation.
5. The Hilbert spaces $L_2(\Omega)$, $H^1(\Omega)$, and $H_0^1(\Omega)$.
6. A geometric interpretation of FEM.
7. A Neumann problem. Natural and essential boundary conditions.
8. Remarks on programming.

Week 2: Abstract formulation of the finite element method for elliptic problems.

1. Introduction. The continuous problem.
2. Discretization. An error estimate.
3. The energy norm.

Weeks 3 & 4: Some finite element spaces and approximation theory for FEM.

1. Introduction. Regularity requirements.
2. Some examples of finite elements.
3. Interpolation with piecewise linear functions in two dimensions.
4. Interpolation with polynomials of higher degree.
5. Error estimates for FEM for elliptic problems.
6. On the regularity of the exact solution.

Week 5: Some applications to elliptic problems.

1. The elasticity problem.
2. Stokes problem.
3. Maxwell's equations.

Weeks 6 & 7: Solving linear systems of equations.

1. Gaussian elimination. Cholesky's method.
2. Condition number.
3. Iterative methods: Jacobi and Gauss-Seidel.
4. The conjugate Gradient method.
5. Preconditioning.
6. Multigrid methods.

Week 8: FEM for parabolic problems.

1. A one dimensional model problem.
2. Semi-discretization in space.
3. Discretization in space and time.
4. Numerical solution of ODEs.
5. The discontinuous Galerkin Method.

Week 9: Boundary element methods.

1. Some integral equations.
   (a) An integral equation for an exterior Dirichlet problem using a single layer potential.
   (b) An exterior Dirichlet problem with double layer potential.
   (c) An exterior Neumann problem with single layer potential.
   (d) Alternative integral equation formulations.
2. Finite element methods.
   (a) FEM for a Fredholm equation of the first kind.
   (b) FEM for a Fredholm equation of the second kind.

Week 11: FEM for hyperbolic problems.

1. Transport in one dimension.
2. The wave equation in one dimension.
3. The wave equation in higher dimensions.
4. A finite element method.
5. Error estimates and adaptive error control.