How to read and learn mathematics - Math as a new language

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During your first several years learning math, and maybe even up to now, you might have been taught math in the following way:

"Here is a way to solve a specific problem (with little or no explanation on why it works or for what purpose*), now repeat this many, many times until you've memorized the method."*

This is *not* how advanced mathematics is learned. You are now being presented with theory which you have to read and understand first. In fact, there exists so much math that it is impossible to show you every single "type" of problem and how to solve it. Also, there are usually many ways to solve a single problem. Math is supposed to be a stimulating and creative subject, and not about tedious repetitions. You should be able to be creative with your solutions!

Of course I am not saying that you shouldn't work any problems. You need to work problems to remember a specific method better, and it can help you understand a certain concept or theorem. Also, it can certainly be helpful to have solved a problem before if you encounter it again, say on a test. But if the textbook or notes had an example for every single type of problem then they would be 20 times as thick!

What we are trying to do is teach you math on a higher level than before. You must adopt a new approach to learning math in order to enhance your performance and understanding. Treat the textbook and lecture notes as your source of information, but also as a dictionary to look up definitions. Indeed, you must realize that you can't just "do" math any more; you also have to read it and think about it. Definitions are important and they have to be internalized before you can really understand anything.

Just like with languages, you must first know what the words mean and then learn how to use them. In math, the words are the concepts or symbols. Theorems prove (i.e. show) relationships between concepts and can give us insight into those concepts and their usefulness. Theorems and facts also give us tools to solve other problems, and often one theorem can be used to solve several problems.

It is important to understand the difference between definitions, theorems, examples and problems. Some students see a definition and wonder *"What am I supposed to solve or do?"* But definitions are not problems to be solved! They are to be read over and over again until you've learned them and understood what they really mean. This has to be clear and takes both time and effort, but it is the only way to make real progress.

It is very important that you treat the examples in the textbook or notes as something which helps you understand what a definition means or how a theorem works. Examples can also help you understand relationships between concepts or show you why a theorem is used and why it works. You must start thinking of examples in this way, and not as a (complete) list of problems which you are supposed to be able to solve by simply repeating them with different numbers. Concentrate on the concepts, because your ultimate goal is to be able to apply them to new situations. When you can do that, then you have truly learned them.

When we see a problem to solve, we have to *first make sure we understand the problem* i.e. know *exactly* what it means by looking up the definitions. Simply reading a problem carefully and thinking about it for a few minutes can make it clearer and thus saves time, which might have been spent in confusion. You'll find that it adds to your enjoyment of the problem. Then, you have to see if you can find some theorems or notes which will help you solve the problem. This approach might be something which you are not familiar with, but it a very effective way to study math. In fact, it is the most successful way to learn by far!

Let's look at an example. Suppose that you are given the following problem:

Show that the vectors $\mathbf{v}_1$*,* $\mathbf{v}_2$*, and* $\mathbf{v}_3$ *are linearly independent, where:*

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}.$$

Now, the thing not to do is mindlessly searching for an identical problem in the textbook or lecture notes. Rather, try to follow these steps:

Step 1 *Identify what you are supposed to do.* Here, show linear independence of vectors.

Step 2 *Look up exactly what that means.* Look up the definition of linear independence. And if you don't know what a vector is, then you must look that up along with the notation used.

Step 3 *See if you can solve the problem directly, i.e. using only the definition.* In this case, the definition of linear independence. You'll find that for vectors to be linearly independent, a certain condition has to be met, which you then have to determine whether is satisfied or not. You can also look for a helpful theorem or comment in the textbook or lecture notes, here involving linear independence of vectors.

Step 4 If you do find something helpful, and there might be many ways to solve the problem, *state and explain what you are doing.* In this case e.g., create a matrix with the vectors as column vectors. The vectors are linearly independent if (and only if) the determinant of the matrix is not zero. So, the problem now involves calculating a determinant. Here, the determinant is 46 which shows that that they are linearly independent.

- Make sure that your solution clearly explains what you are doing.

Now, as a final thought: You might find this confusing or tedious and think that this approach is really only for math majors or someone who is serious about math. The truth is that everyone needs to approach the material this way. Your ultimate goal is to understand the material, and to be able to use it, and present it in a clear way. Focusing on understanding the material will also help you with any other subject you are taking because you'll be able to learn effectively on your own!

Happy studying,
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