Math 8 - Class Work - Solutions

January 25, 2007

Consider the following functional propositions, where the domain of interpretation for all variables is the set S of people in this room.

- $P(x, y) =$ "x knows y's name."
- $Q(x, y) =$ "x and y are friends."
- $R(x) =$ "x owns a cell phone."

Express the following propositions in logical symbols:

1. "There is someone in this room who has no friends."

Answer: $\exists x \forall y\ (\sim Q(x, y))$

Negation: "Everyone has at least one friend" $= \forall x \exists y\ Q(x, y)$

2. "Someone in this room knows everyone's name."

Answer: $\exists x \forall y\ P(x, y)$

Negation: "No one knows everyone's name" $= \forall x \exists y\ (\sim P(x, y))$

3. "Everyone in this room knows somebody's name."

Answer: $\forall x \exists y\ P(x, y)$

Negation: "Someone knows nobody's name." $= \exists x \forall y\ (\sim P(x, y))$

4. "Somebody in this room knows nobody's name besides his/her own."

Answer: $\exists x \forall y\ [(y = x) \vee (\sim P(x, y))]$

Negation: "Everyone knows at least one name besides his/her own"

$$= \forall x \exists y\ [(y \neq x) \wedge P(x, y)]$$

5. "There is someone in this room all of whose friends own cell phones."

Answer: $\exists x \forall y\ (Q(x, y) \Rightarrow R(y))$

Negation: "Everyone has at least one friend who doesn't own a cell phone"

$$= \forall x \exists y\ (Q(x, y) \wedge \sim R(y))$$

6. "Any two friends know each other's names."

Answer: $\forall x \forall y\ [Q(x, y) \Rightarrow (P(x, y) \wedge P(y, x))]$

Negation: "There are two friends who don't know each others' names"

$$= \exists x \exists y \; [Q(x,y) \wedge (\sim P(x,y) \vee \sim P(y,x))]$$

7. "Someone in this room knows the names of everyone who knows his/her name."

Answer: $\exists x \forall y \; (P(y,x) \Rightarrow P(x,y))$

Negation: "Nobody knows the names of everyone who knows his/her name"

$$= \forall x \exists y \; (P(y,x) \wedge \sim P(x,y))$$

8. "Everyone in this room is friends with someone who either does not own a cell phone or has only one friend."

Answer: $\forall x \exists y \; [Q(x,y) \wedge (\sim R(y) \vee (\forall z \; (Q(y,z) \Rightarrow z = x)))]$ (The formula simplifies a little since when x and y are friends, "y has only one friend" is the same as saying "x is the only friend of y.")

Negation: "Someone is only friends with people who own a cell phone and have more than one friend"

$$= \exists x \forall y \; [Q(x,y) \Rightarrow (R(y) \wedge \exists z \; (Q(y,z) \wedge z \neq x))]$$