

Math 8 - Class Work

January 25, 2005

Consider the following functional propositions, where the domain of interpretation for all variables is the set S of people in this room.

- $P(x, y) =$ “ x knows y 's name.”
- $Q(x, y) =$ “ x and y are friends.”
- $R(x) =$ “ x owns a cell phone.”

Express the following propositions in logical symbols:

1. “There is someone in this room who has no friends.”
2. “Someone in this room knows everyone's name.”
3. “Everyone in this room knows somebody's name.”
4. “Somebody in this room knows nobody's name besides his/her own.”
5. “There is someone in this room all of whose friends own cell phones.”
6. “Any two friends know each other's names.”
7. “Someone in this room knows the names of everyone who knows his/her name.”
8. “Everyone in this room is friends with someone who either does not own a cell phone or has only one friend.”

Now practice forming the negations of the above sentences. First try to simplify them as much as possible in English, and then use the formulas

$$\sim (\exists x P(x)) \equiv \forall x (\sim P(x)) \quad \text{and} \quad \sim (\forall x P(x)) \equiv \exists x (\sim P(x))$$

to simplify the negations of your symbolic answers above into expressions that correspond to your negated sentences.