MATHEMATICS 8
TRANSITION TO HIGHER MATHEMATICS
FIRST MIDTERM
October 27, 2006
10 - 10:50 AM

UCSB
Fall 2006
Prof. Sherman

NAME:

UNIVERSITY ID #:

SIGNATURE:

No calculators, books, headphones, notes, or rubber chickens are allowed during this exam.

GRADING:

1:

2:

3:

4:

BONUS:

TOTAL out of 40:

1. a) (2 points each) "If some ducks like reggae, then no cows wear neckties."

Write the converse:

Write the contrapositive:

b) (1 point each) Write these sentences as sentential forms with accurate syntax, using the following letters for atomic propositions:
M = "Mary sings"
B = "Beano cries"
Z = "Zena sleeps"

If Mary doesn't sing, then Beano cries and Zena sleeps. _____

Mary sings only if Beano cries. _____

For Zena to sleep, it is necessary that Mary sing. _____

Either Beano cries or Zena sleeps, but not both. _____

c) (3 points) Which of the following are logically equivalent to $P \Rightarrow Q$?
(1) $\sim (Q \Rightarrow P)$
(2) $(\sim P) \Rightarrow (\sim Q)$
(3) $(\sim Q) \Rightarrow (\sim P)$
(4) $P \wedge \sim Q$
(5) $P \vee \sim Q$
(6) $(\sim P) \wedge Q$
(7) $(\sim P) \vee Q$

Give the numbers of all choices that apply.
You need not show any work. _____

2. (3 points each) TRUE or FALSE. For each statement, either write "TRUE" or "FALSE" and explain why in one or two sentences. In some cases a specific example may suffice.

a) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})(x \leq y)$

b) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x + y < 10)$

c) There are sets A and B such that $A \subseteq B$ and $A \in B$.

d) If $A \subset B$, then $B \neq \varnothing$.

3. (6 points) Use a truth table to decide whether or not the following argument form is valid. EXPLAIN how the truth table justifies your conclusions.

$$\{(P \lor Q) \Rightarrow \sim R, \sim P\} \vDash R \Rightarrow Q.$$

4.a) (2 points) Define the relation $A \subseteq B$, using mathematical/logical notation (no English).

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b) (3 points each) Write the following propositions in mathematical/logical notation (no English).

Every real number has a unique cube root.

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4,559 is not prime.

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If you square a rational number, you get something nonnegative.

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BONUS. (2 points) You are told that the following is a proposition: "If this sentence is true, then this sentence is false."

What can you deduce about its truth value?

(A) It must be true.
(B) It must be false.
(C) It could be either true or false.
(D) It cannot be either true or false (so it's not a proposition).

Give the letter of the correct response here ____, then explain your answer.