Math 8 Midterm Fall 2006

Instructor: G. Wei

Name Solutions

Discussion time (circle one): 8am ; 5pm;

Your Scores: 1. .... **2.** .... **3.** .... **4.** .... **5.** ....

Your Total ....

All questions have equal points. Continue on the back of the page if you need more space. Good luck!

1. Make truth tables for the following propositional forms:

$$P \Rightarrow (Q \vee R), \qquad (P \wedge \sim Q) \Rightarrow R.$$

Then determine if they are equivalent.

Truth Tables

P	Q	R	$Q \vee R$	$\sim Q$	$P \wedge \sim Q$	$P \Rightarrow (Q \vee R)$	$(P \wedge \sim Q) \Rightarrow R$
T	T	T	T	F	F	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	F	T	F	T	T

$P \Rightarrow (Q \vee R)$ and $(P \wedge \sim Q) \Rightarrow R$ have same truth values, so they are equivalent.

2. Negate the following proposition and simplify so that the no negation symbols appear. Use **symbols only**. The universe is all integers.

$$(\forall n)[n > 0 \Rightarrow (\exists a)(\exists b)((a > 0) \wedge (b > 0) \wedge (a^2 + b^2 = n))].$$

This is equivalent to

$$(\forall n)\left[\sim(n > 0) \vee (\exists a)(\exists b)\left((a>0) \wedge (b>0) \wedge (a^2+b^2=n)\right)\right]$$

Its negation is

$$(\exists n)\left[(n>0) \wedge (\forall a)(\forall b)\left((a \le 0) \vee (b \le 0) \vee (a^2+b^2 \neq n)\right)\right]$$

3. Let $\{A_\alpha : \alpha \in \Delta\}$ be an indexed family of sets and B a set. Prove that

$$\left(\bigcap_{\alpha\in\Delta} A_\alpha\right) - B = \bigcap_{\alpha\in\Delta}(A_\alpha - B).$$

Proof: Note that $x \in \left(\bigcap_{\alpha\in\Delta} A_\alpha\right) - B$

$\Leftrightarrow$ $x \in \bigcap_{\alpha\in\Delta} A_\alpha$ and $x \notin B$

$\Leftrightarrow$ $x \in A_\alpha$ for all $\alpha \in \Delta$ and $x \notin B$

$\Leftrightarrow$ $x \in A_\alpha - B$ for all $\alpha \in \Delta$

$\Leftrightarrow$ $x \in \bigcap_{\alpha\in\Delta}(A_\alpha - B)$

Therefore $\left(\bigcap_{\alpha\in\Delta} A_\alpha\right) - B = \bigcap_{\alpha\in\Delta}(A_\alpha - B)$

4. For natural numbers m, n, prove that if mn is odd then both m and n are odd.

Proof: We will prove by contraposition

If m and n are not odd, then either m or n are even. If m is even, then $m = 2k$ for some integer k So $mn = 2kn = 2(kn)$ is even

Similarly if n is even, we get mn is even

By contraposition, if mn is odd, then both m and n are odd.

5. Show that for all natural numbers n,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Proof: When $n=1$, $\frac{1 \cdot (1+1)(2 \cdot 1+1)}{6} = 1 = 1^2$

the statement is true.

Assume it's true for n, we will show it's true for $n+1$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 + (n+1)^2 = [1^2 + 2^2 + 3^2 + \cdots + n^2] + (n+1)^2$$

by induction hypothesis

$$= \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= (n+1)\left[\frac{n(2n+1) + 6(n+1)}{6}\right]$$

$$= (n+1)\frac{(n+2)(2n+3)}{6} = \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}$$

Therefor it's true for $n+1$. By PMI

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{for all } n \in \mathbb{N}$$