Math 8 - Homework #5

Due: May 5, 2009

1. Express each of the following statements using sets. Your answers should be of the form "[something] $\in$ (or $\notin$) [some set]".

   (a) x is a nonnegative integer that is smaller than 5.

   (b) Either a or b equals 1.

   (c) Neither x nor y is 0.

2. Write each of the sets below in two ways: a) in the form $\{x \in U \mid P(x)\}$, and b) in the form $\{f(x) \mid x \in S\}$ where $f(x)$ is a function (possibly of multiple variables), and S and U are some sets.

   (i) $A = \{1, 2, 4, 8, 16, \ldots\}$ is the set of all (integer) powers of 2.

   (ii) B is the set of all integers that can be written as the sum of two perfect squares.

   (iii) C is the set of all the reciprocals of natural numbers.

3. (a) Prove that $\{2k - 1 \mid k \in \mathbb{Z}\} = \{2k + 1 \mid k \in \mathbb{Z}\}$.

   (b) Are the sets $\{2k - 1 \mid k \in \mathbb{N}\}$ and $\{2k + 1 \mid k \in \mathbb{N}\}$ also equal? Justify your answer. (Suggestion: start listing the elements in these sets by plugging in different natural numbers for k.)

4. **Exercises 2.1 p. 76-77:** 5)b; 13; 19)a–d.

5. **Exercises 2.2 p. 83-84:** 2)d, f; 10)f (it may help to draw a Venn diagram); 12)b,c (you may draw a Venn diagram);

6. **Exercises 2.3 p. 92-93:** 1)d, j, m.