Math 8 - Class Work

10 April 2009

1. **Every/Only.** Sometimes sentences with the words "only" and "every" can be conditional statements in disguise. For example, "Every even number is a multiple of two." can be rephrased as "If a number is even, then it is a multiple of two."

   Rewrite the following propositions as conditional statements in the form "If ..., then ...".

   (a) Every small dog is a puppy.

   (b) Everybody who speaks Spanish also speaks Italian.

   (c) I eat all vegetables.

   Similarly, for "only", the sentence "I only eat spaghetti." can be rephrased as "If it is not spaghetti, then I will not eat it." or "If I eat something, then it is spaghetti." Notice that the second rephrasing is just the contrapositive of the first.

   Rewrite the following as conditional statements in the form "If ..., then ..." in two equivalent ways (i.e., your two sentences should be the contrapositives of one another).

   (d) I only have class on Fridays.

   (e) Only squirrels have fluffy tails.

   (f) It only rains in the mornings.

2. (Optional, but recommended) You may have noticed that the word "only", like the word "unless" can be ambiguous. For example, does "I only eat spaghetti." really mean the same thing as "If I eat something, then it is spaghetti."? Or does it mean "I eat something if and only if it is spaghetti."?

   Think about this question for (d)-(f) above. In each case, which interpretation of "only" seems more appropriate? Also, how does the meaning of "I only eat spaghetti." compare to the meaning of "I will not eat it, *unless* it is spaghetti." ? Can you rewrite the sentences (d)-(f) using "unless"? Finally, try to decide whether the ambiguity with "only" is equivalent to the ambiguity with "unless".

3. Consider the following functional propositions, where the domain of interpretation for all variables is the set S of people in this room (i.e., "Someone" should be read as "Someone in this room", etc.).

   - $P(x, y) =$ "x knows y's name."
   - $Q(x, y) =$ "x and y are friends."
   - $R(x) =$ "x owns a car."

   Express the following propositions in logical symbols:

   (a) "There is someone in this room who has no friends."

   (b) "Someone in this room knows everyone's name."

   (c) "Everyone in this room has a friend who owns a car."

   (d) "Any two friends know each other's names."

   (e) "Anybody who owns a car is friends with everyone."

   (f) "Someone in this room knows the names of everyone who knows his/her name."

   Now practice forming the negations of the above sentences. First try to simplify them as much as possible in English, and then use the formulas

   $$\sim (\exists x\ P(x)) \equiv \forall x\ (\sim P(x)) \quad \text{and} \quad \sim (\forall x\ P(x)) \equiv \exists x\ (\sim P(x))$$

   to simplify the negations of your symbolic answers above into expressions that correspond to your negated sentences.