11/21/07 Math 8 - Lecture Notes.

**Recall** 2 sets $A$ & $B$ have the same cardinality if there exists a bijection $f: A \to B$. (write: $A \approx B$)

**more examples**

1) consider $f: \mathbb{Z} \to \mathbb{Z}$ given by $f(n) = 2n$.

$f$ is one-to-one : If $f(n) = f(m)$, then $2n = 2m$. Thus $n = m$.

$f$ is not onto:

$\mathrm{Im}(f) = \{2n \mid n \in \mathbb{Z}\}$ = set of even integers $= 2\mathbb{Z}$.

Thus $f: \mathbb{Z} \to 2\mathbb{Z}$ is a bijection.

& $\mathbb{Z} \approx 2\mathbb{Z}$.

2) consider $f: \mathbb{N} \to \mathbb{Z}$ given by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{if } n \text{ is odd} \\ -\frac{n}{2}, & \text{if } n \text{ is even.} \end{cases}$$

$f$ is one-to-one: Suppose $f(a) = f(b)$ for $a, b \in \mathbb{N}$.

If $f(a) \geq 0$ then $a, b$ are odd.

$\Rightarrow \frac{a-1}{2} = \frac{b-1}{2} \Rightarrow a = b$.

If $f(a) < 0$, then $a, b$ are even

$\Rightarrow -\frac{a}{2} = -\frac{b}{2} \Rightarrow a = b$.

$f$ is onto: Let $n \in \mathbb{Z}$. if $n < 0$, $n = -\left(\frac{-2n}{2}\right) = f(-2n)$

if $n \geq 0$, $n = \frac{(2n+1)-1}{2} = f(2n+1)$.

$\therefore \mathbb{N} \approx \mathbb{Z}$.

1 2 3 4 5 ...

... -2 -1 0 1 2 3 4 ...

**Definition** For an integer $n \geq 0$, let

$\mathbb{N}_n = \{1, 2, 3, \ldots, n\} = \{x \in \mathbb{N} \mid 1 \leq x \leq n\}$.

A set $S$ is finite if $\exists n \geq 0$ with $S \approx \mathbb{N}_n$.

Write $|S| = n$. $S$ is infinite if it is not finite.