

11/21/07 Math 8 - Lecture Notes.

Recall 2 sets A & B have the same cardinality if there exists a bijection $f: A \rightarrow B$.
(write: $A \approx B$)

more examples

1) consider $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(n) = 2n$.
 f is one-to-one: if $f(n) = f(m)$, then
 $2n = 2m$.
Thus $n = m$.

f is not onto:
 $\text{Im}(f) = \{2n \mid n \in \mathbb{Z}\} = \text{set of even integers} = 2\mathbb{Z}$.

Thus $f: \mathbb{Z} \rightarrow 2\mathbb{Z}$ is a bijection.
 $\mathbb{Z} \approx 2\mathbb{Z}$.

2) consider $f: \mathbb{N} \rightarrow \mathbb{Z}$ given by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{if } n \text{ is odd} \\ -\frac{n}{2}, & \text{if } n \text{ is even.} \end{cases}$$

f is one-to-one: Suppose $f(a) = f(b)$ for $a, b \in \mathbb{N}$.

if $f(a) \geq 0$ then a, b are odd.

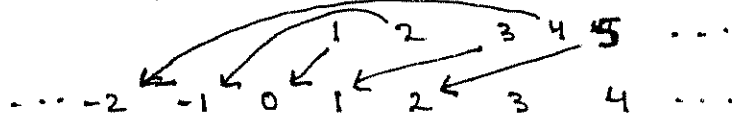
$$\Rightarrow \frac{a-1}{2} = \frac{b-1}{2} \Rightarrow a = b.$$

if $f(a) < 0$, then a, b are even

$$\Rightarrow -\frac{a}{2} = -\frac{b}{2} \Rightarrow a = b.$$

f is onto: Let $n \in \mathbb{Z}$. if $n < 0$, $n = -\left(\frac{-2n}{2}\right) = f(-2n)$
if $n \geq 0$, $n = \frac{(2n+1)-1}{2} = f(2n+1)$.

$\therefore \mathbb{N} \approx \mathbb{Z}$.



Definition For an integer $n \geq 0$, let

$$\mathbb{N}_n = \{1, 2, 3, \dots, n\} = \{x \in \mathbb{N} \mid 1 \leq x \leq n\}.$$

A set S is finite if $\exists n \geq 0$ with $S \approx \mathbb{N}_n$.

Write $|S| = n$. S is infinite if it is not finite.