Math 8 - Midterm 1 Review Problems
Fall 2007

1. Consider the proposition

   "If I have either both a fish and a cat or both a cat and a dog, then I do not have a fish or a dog."

   (a) Rewrite the proposition symbolically as a statement form in terms of statement variables P, Q and R (you may use different letters if you wish). Be sure to say which propositions are represented by your variables.

   (b) Construct a truth table for your answer to (a).

   (c) By looking at the truth table, find a simpler statement form in the same variables that is logically equivalent to this proposition. (Tricky, but good practice.)

   (d) Convert your answer to (c) back into an English sentence.

2. **True or False.** If the statement is True, give a proof. If it is False, give a reason why or a counterexample.

   (a) $a \in \mathbb{Z} \wedge b \in \mathbb{Z} \Leftrightarrow a + b \in \mathbb{Z}$

   (b) $Q \Rightarrow [P \wedge ({\sim} Q \vee {\sim} P)]$ is a tautology.

   (c) If a perfect square is prime, then it is divisible by all primes.

3. Consider the implication "If it rains sometimes, then nobody is happy." Write English sentences for (a) the converse; (b) the contrapositive; and (c) the negation of this statement. (Your answers should be as simple and natural as possible, so you should avoid phrases like "It is not the case that..." or "It is false that...", etc.)

4. Consider the proposition: "Every nonzero rational number is equal to a product of two irrational numbers."

   (a) Write this proposition as a conditional (If..., then...) statement.

   (b) Now write it using only symbols and no words. (You may use the symbols for the sets of rational numbers, etc. from class. You do not need to worry about quantifiers here.)

   (c) Prove this proposition.

5. In this problem x and y are real numbers. Consider the proposition: "If the product xy is irrational, then either x or y is irrational."

   (a) State the contrapositive of this proposition.

   (b) Prove this proposition.

   (c) Is the converse of this proposition also true (for all real numbers x, y)? Explain.

6. Which (if any) of the sets $A = \{1, 2, \{4, 3\}, 5\}$, $B = \{\{3, 4\}, 5, 1, 2\}$, $C = \{\{1, 2\}, 4, 3, 5\}$ are equal to each other?

7. Express the following sets in set-builder notation, i.e., $A = \{x \in S \mid P(x)\ \}$.

   (a) $A = \{0, 1, 4, 9, 16, \ldots\}$ (ie., the set of perfect squares).

   (b) B is the interval $(-1, 2]$ on the real number line.

   (c) $C = \{9, 19, 29, \ldots\}$ is the set of all positive integers that end in 9. (This can be done without using any words.)

8. Prove that the two sets $A = \{x \in \mathbb{R} \mid x^2 \in \mathbb{Q}\}$ and

$$B = \{x/y \mid x, y \in \mathbb{R} \land y \neq 0 \land x^2, y^2 \in \mathbb{Z}\ \}$$

   are equal.