Math 8 - Homework #4

Due: October 25, 2007

1. Express each of the following statements using sets. Your answers should be of the form "[something] $\in$ (or $\notin$) [some set]".

   (a) x is a nonnegative integer that is smaller than 5.

   (b) Either a or b equals 1.

   (c) Neither x nor y is 0.

2. Describe the sets from problem 2, parts (a)-(d), on page 47 of the text in the form $\{f(x) \mid x \in S\}$, where $f(x)$ is a function, and S is some set.

3. (a) Prove that $\{2k-1 \mid k \in \mathbb{Z}\} = \{2k+1 \mid k \in \mathbb{Z}\}$.

   (b) Are the sets $\{2k-1 \mid k \in \mathbb{N}\}$ and $\{2k+1 \mid k \in \mathbb{N}\}$ also equal? Justify your answer. (Suggestion: start listing the elements in these sets by plugging in different natural numbers for k.)

4. (optional) In class, we wrote the set of even integers as $2\mathbb{Z} = \{2k \mid k \in \mathbb{Z}\}$. In this exercise, we explore the arithmetic of sets a little more. All sets considered here will be subsets of $\mathbb{R}$, meaning that all their elements are assumed to be real numbers.

   (a) If we replace $\mathbb{Z}$ with $\mathbb{R}$ in the above example, what set do we get? In other words, describe the set $2\mathbb{R}$.

   (b) Let $m, n \in \mathbb{Z}$. The set of multiples of n can be written $n\mathbb{Z} = \{nk \mid k \in \mathbb{Z}\}$. We can also write $m\mathbb{Z} + n\mathbb{Z} = \{mx + ny \mid x, y \in \mathbb{Z}\}$ for the set of all sums of multiples of m and n. Describe the following sets: (i) $2\mathbb{Z} + 3\mathbb{Z}$; (ii) $2\mathbb{Z} + 4\mathbb{Z}$; (iii) $2\mathbb{N} + 3\mathbb{N}$. (Suggestion: start by listing some elements of these sets by choosing different values for x and y in the expression $mx + ny$.)