Arithmetic Review Sheet
Math 8 - Fall 2007

The following are basic definitions and facts from arithmetic, most of which are probably already familiar to you. As we will be doing many examples in class dealing with these concepts, it is important that you review and remember them.

- An **integer** is a whole number: $\ldots, -1, 0, 1, 2, \ldots$. The *set* of all integers is denoted $\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$.
- A **natural number** is a positive integer: $1, 2, 3, \ldots$. The *set* of all natural numbers is denoted $\mathbb{N} = \{1, 2, 3, \ldots\}$.
- A **rational number** is a fraction of integers; i.e., p/q where p and q are integers and $q \neq 0$. The set of all rational numbers is denoted $\mathbb{Q}$.
- A **real number** is any number that can be expressed using decimals. These correspond precisely to the points on the real-number line (x-axis). The set of all real numbers is written $\mathbb{R}$.
- A real number is **irrational** if it is not rational. The set of irrational real numbers can be expressed as $\mathbb{R} \setminus \mathbb{Q}$ or $\mathbb{R} - \mathbb{Q}$.
- We say that an integer n is a **perfect square** if it is the square of another integer m. Thus the set of perfect squares is $\{0, 1, 4, 9, 16, 25, 36, \ldots\}$.
- For integers a and b, we say that a is a **multiple** of b if $a = bc$ for some integer c. When this happens, we also say that b **divides** a, or that b is a **divisor** or **factor** of a. Symbolically, we write this as $b|a$.

  For example, an integer n is **even** if $2|n$; i.e., $n = 2k$ for some integer k. An integer n is **odd** if it is not even; i.e., if $n = 2k + 1$ for some integer k.
- A natural number n is **prime** if its only positive factors are 1 and n. If n is not prime, then we say it is **composite**. For example, the first several prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots$. Also, 191 is prime, but $195 = 3 \cdot 5 \cdot 13$ is composite.
- **The Fundamental Theorem of Arithmetic** Any natural number $n > 1$ can be factored into a product of powers of primes

  $$n = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r},$$

  where $p_1, p_2, \ldots, p_r$ are distinct primes and $e_1, e_2, \ldots, e_r$ are positive integers.

  Moreover any such factorization is unique up to the order of factors. This means that if n has another factorization into positive powers of distinct primes $n = q_1^{f_1} \cdots q_s^{f_s}$, then $r = s$ and after a suitable reordering of the factors, $p_i = q_i$ and $e_i = f_i$ for all i in the range $1 \leq i \leq r$.

- The **least common multiple** of two positive integers a and b is the smallest natural number c such that c is a multiple of both a and b (i.e., c is the smallest number such that $a|c$ and $b|c$). Such a c may be denoted $\mathrm{lcm}(a, b)$.

  The **greatest common divisor** of two positive integers a and b is the largest natural number d that divides both a and b (i.e., d is the largest number such that $d|a$ and $d|b$). Such a d may be denoted $\gcd(a, b)$.

  Two positive integers a and b are said to be **relatively prime** if $\gcd(a, b) = 1$; i.e., they have no common prime factor.

- **Challenging Exercise.** Let a and b be positive integers. Prove that

$$\mathrm{lcm}(a, b) \cdot \gcd(a, b) = a \cdot b.$$

  (Hint: Use the fundamental theorem of arithmetic.)