

Final Exam
Math 5C, UCSB, Fall '06

You have 3 hours to complete this exam.

Name: _____

Perm #: _____

Signature: _____

Discussion section: _____

Show all your work. Partial credit will be given only if work is relevant and correct. *Please make your work as clear and easy to follow as possible.* You might want to put scratch work on the back of every sheet, and put neat clean work on the front of every sheet. You don't need to simplify your answers but you need to justify them.

Problem	Possible Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Extra	10	
Total	50 (+10)	

Name: _____

Exercise 1.

Verify Stoke's theorem for the vector field $\mathbf{F} = (xy, xz, z + 1)$ where S is the surface defined by $z = x^2 + y^2$ and $z = 1$.

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Exercise 2.

Find the Fourier series of the function

$$f(x) = \begin{cases} 2 & \text{if } 2k\pi \leq x < (2k+1)\pi \\ 0 & \text{if } (2k+1)\pi \leq x < (2k+2)\pi \end{cases}$$

where $k = 0, \pm 1, \pm 2, \dots$

$f(x) =$ _____

Name: _____

Exercise 3.

Let S be the upper half surface of the sphere $x^2 + y^2 + z^2 = 4$. Compute

$$\int \int_S z^2 dS = \underline{\hspace{15em}}$$

Name: _____

Exercise 4.

Expand the function $f(x) = x^2 e^{2x}$ into a power series around the point $x = 0$.

Name: _____

Exercise 5.

Determine whether the following series converge (Justify your answer):

$$1) \sum_{k=1}^{\infty} \frac{(-1)^k}{2k+1}$$

$$2) \sum_{k=1}^{\infty} \frac{e^k}{e^{2k}-1}$$

$$3) \sum_{k=1}^{\infty} \frac{1}{2k^2-1}$$

Name: _____

Exercise 6.

Find the solution for the wave equation:

$$\begin{cases} u_{tt} - 25 u_{xx} = 0 & t \geq 0 \\ u(0, t) = 0 & t \geq 0 \\ u(\pi, t) = 0 & t \geq 0 \\ u(x, 0) = 2 \sin(x) & 0 \leq x \leq \pi \\ u_t(x, 0) = 3 \sin(2x) & 0 \leq x \leq \pi \end{cases}$$

$u(x, t) =$ _____

Name: _____

Exercise 7.

Determine for which x the following series converges:

$$\sum_{k=1}^{\infty} e^{kx}$$

Compute its value for all the values of x , for which it converges.

Name: _____

Exercise 8. (Extra)

Let S be the surface defined by $z = x^2 + y^2$, $z = -x^2 - y^2$ and $x^2 + y^2 = 1$ and let $\mathbf{F} = (x + y, y + z, x^2 + z)$.

Compute

$$\int \int_S \mathbf{F}_n \, d\sigma = \underline{\hspace{10em}}$$

Name: _____