

## Math 5C, Midterm 2 Review Problems

Winter 2007

1. The first several terms of a sequence are given. Find a formula in terms of  $n$  for the  $n^{\text{th}}$  term of the sequence (be sure to say what value  $n$  starts at). Then find the limit of the sequence as  $n$  tends towards infinity, if one exists.

(a)  $-4, 1, \frac{-1}{4}, \frac{1}{16}, \dots$

(b)  $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \dots$

(c)  $1, -3, 8, -24, 35, -48, 63, \dots$

(d)  $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \frac{9}{16}, \dots$

2. Find the sum of the infinite series, or show that it diverges:

(a)  $\sum_{n=1}^{\infty} 2^{1-2n}$

$\sum_{n=1}^{\infty} \left( \frac{\sqrt{2n+1}}{n} - \frac{\sqrt{2n+3}}{n+1} \right)$

3. Do the following series converge or diverge? Justify your answers.

(a)  $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{3n^2}}$

(b)  $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(c)  $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

(d)  $\sum_{n=1}^{\infty} \frac{n^n}{2 \cdot 4 \cdot 6 \cdots (2n)}$

4. Find the interval of convergence of the following power series:

(a)  $\sum_{n=0}^{\infty} \frac{(x-3)^n}{4^n}$

$\sum_{n=0}^{\infty} \frac{x^{2n}}{n+1}$

5. Suppose that the power series  $\sum_{n=0}^{\infty} c_n(x-1)^n$  converges when  $x=2$  and diverges when  $x=-3$ . For each of the following values of  $x$ , state whether the series converges or diverges, or whether you can't tell: (a)  $x=1$ ; (b)  $x=0$ ; (c)  $x=1/2$ ; (d)  $x=3$ ; (e)  $x=5$ ; (f)  $x=6$ .

6. Find the MacLaurin series and its interval of convergence for the function  $f(x) = \frac{x^2}{(1+x)^2}$ .

7. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{n}{2^n}$ . (Hint: This sum can be realized as the MacLaurin series of some function  $f(x)$  evaluated at a certain value of  $x$ .)