Math 5C, Final Exam Review Problems

Winter 2007

The Final Exam will cover material from the entire course, including several problems on vector calculus, and several on sequences and series. You should thus also review old homework problems from Chapters 5 and 6, along with your previous midterms and previous review problems. Roughly half the test will be on new material (Ch. 6.12-6.13, 7.1-7.5, 7.17, 10.7). The most important topics from previous chapters are the following

- Line Integrals (Ch. 5.8)
- Surface Integrals (Ch. 5.9-5.10)
- Divergence Theorem (Ch. 5.11) and Triple Integrals
- Stokes' Theorem (Ch. 5.12)
- Convergence/Divergence tests for series (Ch. 6.6-6.7)
- Radius/Interval of Convergence of Power Series (Ch. 6.11, 6.15)
- Taylor and Maclaurin Series (Ch. 6.16)

1. Let $f(x) = \frac{1}{4+x}$.
   (a) Find the Maclaurin series for $f(x)$, and compute its radius of convergence.
   (b) Find the Taylor series for $f(x)$ centered at $x = 1$, and compute its radius of convergence.
   (c) Find the Taylor series for $g(x) = \ln(4 + x)$ centered at $x = 1$, and compute its radius of convergence.

2. For what values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^x}$ converge? (Bonus: Does it converge uniformly on this entire set? If not, is there a smaller set on which it does?)

3. Show that the series of functions $\sum_{n=1}^{\infty} ne^{-nx}$ converges uniformly on $[1/2, \infty)$.

4. Let $f(x) = \begin{cases} 1, & \text{if } -\pi/2 \le x \le \pi/2 \\ -1, & \text{if } -\pi \le x < -\pi/2 \text{ or } \pi/2 < x \le \pi \end{cases}$
   (a) Find the Fourier series for $f(x)$, and sketch its graph.
   (b) Use part (a) (or other methods) to find the Fourier series for

$$g(x) = \begin{cases} -\pi - x, & \text{if } -\pi \le x < -\pi/2 \\ x, & \text{if } -\pi/2 \le x \le \pi/2 \\ \pi - x & \text{if } \pi/2 < x \le \pi \end{cases}$$

(c) Show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$. (You should practice using one of the Fourier series above, or else one from lecture, rather than a power series.)

5. Find the Fourier series for $f(x) = e^x$. (The computations are simpler if you use the complex form: the integral of e^{cx} is still e^{cx}/c, even if c is a complex number.)

6. Find the solution of the wave equation $\dfrac{\partial^2 u}{\partial t^2} - 4\dfrac{\partial^2 u}{\partial x^2} = 0$ with initial displacement given by $f(x) = 0$, and initial velocity given by $g(x) = \sin^2 x$. (Hint: use a half-angle formula!)

   Sketch the solution when $t = \pi/2$.