Math 5C, Homework Problems from Lecture
Due: 7 Feb. 2007

These problems complete the proof of Gauss's Law from lecture (1/30). You may print out this page and write your solutions on it.

1. Consider the vector field
$$\mathbf{F} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{3/2}},$$
   which is differentiable on the domain $D = \mathbb{R}^3 - \{(0,0,0)\}$.

   (a) Show that $\operatorname{div}(\mathbf{F}) = 0$ on D.

   (b) If S is a sphere of radius r centered at the origin and with outer normal $\mathbf{n}$, show that
$$\int\int_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = 4\pi.$$
   (Notice that on S, $\mathbf{F}$ simplifies to $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})/r^3$.)