Math 5B, Final Review Topics and Problems

Fall 2006

Here is a brief list of the key topics and important formulas we have covered since the last midterm. You should know how to do each thing listed and/or know the definitions of the key concepts. The numbers in parantheses refer to the numbering system in the text. **These are the most important formulas/facts/definitions that you should have memorized and understand how to use.** You should also review the two midterms and the review problems for them.

- Ch. 4.1 : Review basic integration techniques from single variable calculus: u-substitution, integration by parts, trig substitution, trig integrals and trig identities.
- Ch. 4.3: Double Integrals and Iterated Integrals: (4.33), (4.34). Volume under a surface (4.35). Area of a region (4.36). Changing the order of integration (ex. 5, p. 235).
- Ch. 4.6: Change of Variables in Double Integrals: (4.61). Change of Variables to Polar Coordinates: (4.64).
- Ch. 4.7: Arc Length: (4.69), (4.70), and (4.71). Surface Area: (4.72).
- Ch. 5.2: Line Integrals in the Plane: (5.4), (5.5).
- Ch. 5.3: Line Integrals with respect to Arc Length: (5.12). Properties of Line Integrals: (5.18-5.22). Line Integrals to calculate Area: (5.24).
- Ch. 5.4: Line Integrals in Terms of Vectors: (5.26) and (5.31) using tangential components, and (5.38) using normal components.
- Ch. 5.5: Green's Theorem: (5.40). Vector Interpretation of Green's Theorem: (5.43-5.44).
- Ch. 5.6: Independence of Path: Theorem I (5.46), (5.48). Theorems II (5.51) and III (5.52).

1. Integrate $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$.
2. Integrate $\int\int_R \frac{1}{1+x^2+y^2}\, dx\, dy$ where R is the region bounded by the top half of the unit circle and the x-axis.
3. Integrate $\int\int_R 8xy\, dx\, dy$ where R is the interior of the rectangle with vertices $(0,0)$, $(1,1)$, $(2,-2)$ and $(3,-1)$.

4. Consider the surface $z = \frac{2}{3}(x^{3/2}+y^{3/2})$ above the triangle R with vertices $(0,0), (1,0), (0,1)$ in the xy-plane.

   (a) Find the volume of the region below the surface and above the triangle R.

   (b) Find the surface area of the surface above the triangle R.

5. Find the area inside the closed curve C with parametric equations $x(t) = (t-t^2)\cos(\pi t)$ and $y(t) = (t-t^2)\sin(\pi t)$ for $0 \le t \le 1$.

6. Evaluate the line integrals.

   (a) $\displaystyle\int_C xy^4 \, ds$ where C is the top half of the circle $x^2 + y^2 = 4$ from $(2,0)$ to $(-2,0)$.

   (b) $\displaystyle\int_C \mathbf{u} \cdot d\mathbf{r}$ where $\mathbf{u} = \dfrac{x}{y}\mathbf{i} + \dfrac{y}{x}\mathbf{j}$ and C is the (shorter) arc of the unit circle from $(\sqrt{3}/2, 1/2)$ to $(1/2, \sqrt{3}/2)$.

7. Evaluate $\displaystyle\oint_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ when (a) C is the unit circle traversed in the counter-clockwise direction; and (b) C is the parallelogram with vertices $(2,3), (3,5), (5,2), (6,4)$ traversed in the counter clockwise direction.

8. Evaluate $\displaystyle\oint_C y^3\,dx - x^3\,dy$ where (a) C is the unit circle traversed counter-clockwise; and (b) C is the square with vertices $(\pm 1, \pm 1)$ traversed clockwise.

9. Evaluate the following integrals.

   (a) $\displaystyle\int_C \frac{3x^2}{y}\,dx - \frac{x^3}{y^2}\,dy$ where C is the parabola $y = 2 + x^2$ from $(0,2)$ to $(1,3)$.

   (b) $\displaystyle\int_C \sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy$ where C is the curve $y = 16x^3/\pi^2$ from $(0,0)$ to $(\pi/4, \pi/4)$.

   (c) $\displaystyle\oint_C [\sin(xy) + xy\cos(xy)]\,dx + x^2\cos(xy)\,dy$ where C is the unit circle in the counter-clockwise direction.

   (d) $\displaystyle\oint_C xy^6\,dx + (3x^2y^5 + 6x)\,dy$ where C is the ellipse $x^2 + 4y^4 = 4$ traversed in the counter-clockwise direction. (Hint: the area of the ellipse $x^2/a^2 + y^2/b^2 = 1$ is $ab\pi$.)