Math 5B, Midterm 2 Review Problems

Fall 2006

1. (a) Convert the point $(1, -1, 1)$ from rectangular to cylindrical coordinates.
   (b) Convert $(2, \pi/2, 2\pi/3)$ from spherical to rectangular coordinates.

2. Suppose z and w are functions of x and y given by the equations $z = \dfrac{1+y}{y-x} + 2$ and $w = e^{x+2y} - 1$. Find the Jacobian matrix of the inverse mapping when $(z, w) = (2, 0)$, and simplify your answer.

3. The two equations $xy + uv = 1$ and $xu + yv = 1$ define u and v implicitly as functions of x and y.
   (a) Find the Jacobian matrix $\frac{\partial(u,v)}{\partial(x,y)}$.
   (b) Calculate $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial x \partial y}$.

4. Let S be the surface given by the equation $x^3 - xy - yz - xz - x + 2 = 0$.
   (a) Show that the curve C whose equation is $\mathbf{r}(t) = \left\{ \begin{array}{lcl} x & = & t+1 \\ y & = & t^2 \\ z & = & 2 \end{array} \right.$ is contained in the surface S.
   (b) Find the equation of the tangent line to C at the point $(2, 1, 2)$.
   (c) Find the equation of the tangent plane to S at the point $(2, 1, 2)$.

5. Find all critical points of the function $f(x, y) = 3x^3 - 6xy + y^2$, and classify each as a relative min, relative max, or saddle point.

6. Suppose you want to construct a rectangular wooden box without a top so that the volume is 32 cubic feet. What dimensions (x = length, y = width, z = height) of the box will minimize the amount of wood you need to construct it?

7. Let $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ be a vector field on $\mathbb{R}^3$.

   (a) Find curl($\mathbf{v}$).

   (b) Show that there is no differentiable vector field $\mathbf{u}$ on $\mathbb{R}^3$ such that $\mathbf{v} = \text{curl}(\mathbf{u})$.

8. Let $\mathbf{v} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k}$ be a vector field on the surface S defined by the equation $x^2 + y^2 + z^2 = 1$ (i.e., S is the unit sphere in $\mathbb{R}^3$).

   (a) Show that at each point of S, the vector field $\mathbf{v}$ is tangent to S.

   (b) Is $\mathbf{v} = \nabla f$ for some differentiable function $f(x, y, z)$? Justify your answer.

   (c) Is $\mathbf{v} = \text{curl}(\mathbf{u})$ for some differentiable vector field $\mathbf{u}$ on S? Justify your answer.