# Math 5B, Midterm 1 Review Problems

Fall 2006

1. Consider the two planes $P_1$, defined by the equation $2x - y + z = 1$, and $P_2$, given by the equation $-x + y - z = 3$.

   (a) Find parametric equations for the line that is the intersection $P_1 \cap P_2$ of the planes $P_1$ and $P_2$.

   (b) Does there exist a line that is perpendicular to both planes $P_1$ and $P_2$? Justify your answer.

   (c) Find an equation of the plane that contains the point $(1, 1, 1)$ and the line $L$ with equations $x(t) = 1 + 2t,\ y(t) = 3 - t,\ z(t) = 4t$.

2. Sketch at least 5 level curves of the surface given by the equation $z = e^{x-y}$, and then sketch the surface. (Be sure to label your axes.)

3. Calculate the following limits, or show that they do not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1 + xy}{x^2 + y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \left(\frac{x}{y} + \frac{y}{x}\right)$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2} - 1}{x^2 + y^2}$

4. Let $f(x, y) = \dfrac{2}{\frac{1}{x} + \frac{1}{y}}$.

   (a) What is the domain $D$ of $f$? (Write your answer in the form $\{(x, y) \in \mathbb{R}^2 \mid \ldots\}$.)

   (b) Find $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in D}} f(x, y)$.

   (Note: To compute this limit, you should only consider points $(x, y)$ that are in the domain $D$ of $f$, and not any points $(x, y)$ near $(0, 0)$ where $f(x, y)$ is undefined.)

5. Let $z = \frac{x-y}{x^2+y^2}$, and let $x = r\cos\theta$ and $y = r\sin\theta$.

(a) Find $dz$ in terms of $dx$ and $dy$.

(b) Find $dz$ in terms of $dr$ and $d\theta$. (Your answer should not contain any $x$'s or $y$'s.)

(c) Find $(\frac{\partial z}{\partial r})_\theta$ and $(\frac{\partial z}{\partial r})_x$ (for the second, assume $y > 0$).

(d) Approximate the value of $z$ (using (a)) when $x = 1.01$ and $y = 0.98$.

6. Suppose we have functions $\mathbf{z}(y_1, y_2, y_3) : \mathbb{R}^3 \to \mathbb{R}^3$ and $\mathbf{y}(x_1, x_2) : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\mathbf{z} = \begin{cases} z_1 &= y_1 y_2 - 2y_3 e^{y_2} \\ z_2 &= y_2 \cos y_1 - y_1 \sin y_2 + y_1 y_2 y_3^2 \\ z_3 &= y_1^3 + y_2^3 + y_3^3 \end{cases} \quad \text{and} \quad \mathbf{y} = \begin{cases} y_1 &= x_1^2 - x_2^2 \\ y_2 &= 2x_1 x_2 \\ y_3 &= x_1 + x_2 \end{cases}$$

Write the Jacobian matrix of the composition $\mathbf{z} \circ \mathbf{y}$ as a product of two matrices (do not evaluate this product), and compute $\frac{\partial z_2}{\partial x_1}$.

7. Let $f(u)$ be a function of a single variable $u$, and define $z(x, y) = f(ax + by)$ where $a$ and $b$ are fixed real numbers. Show that

$$b\frac{\partial z}{\partial x} - a\frac{\partial z}{\partial y} = 0.$$

8. Suppose a function $f(x, y)$ of two variables satisfies the law $f(tx, ty) = tf(x, y)$ for all values of $x, y$ and $t$ in $\mathbb{R}$. Show that $xf_x + yf_y = f(x, y)$ for all values of $x$ and $y$.