Math 3B, Final Exam Review
Spring 2007

The problems on the final will be similar to the ones from the homework assignments, quizzes and midterms. The exam will be about twice as long as the midterms and will emphasize the newer material covered since the last midterm (sections 6.1-6.5, 8.1-8.3). However, it will be cumulative and may include questions on any of the material covered this quarter (sections 4.10, 5.1 - 5.5, 7.1-7.5, 7.8). Also, be advised that to do many of the problems on the new material, you will need to evaluate integrals and that requires knowledge of integration techniques, etc. Below is an outline of the different topics you should know, along with lists of practice problems from Stewart. This list is not exhaustive: only the most important topics from earlier sections have been included. The midterm review sheets provide information on additional topics.

1. **Areas of Regions (6.1, 5.2).** Calculate the area between two curves $y = f(x)$ and $y = g(x)$. Often this requires finding where the graphs intersect by solving $f(x) = g(x)$ for x. But be careful, drawing an acurate picture of the region can be crucial. (**6.1** # 9, 13)

2. **Volumes of Revolution.** If R is a region in the xy-plane, calculate the volume of the solid obtained when R is rotated about one of the axes or a line parallel to one of the axes. (**6.2** # 3, 9, 25, 27, 29, 31, 35, **6.3** # 5, 9, 19)

   (a) **Disk Method (6.2).** $V = \int_a^b \pi r^2 \, dx$. Use this method when R is the region under the graph of $y = f(x)$, and it is being rotated about a HORIZONTAL line. If the axis of revolution is $y = c$, then $r = f(x) - c$.

   (b) **Washer Method (6.2).** $V = \int_a^b \pi(r_2^2 - r_1^2) \, dx$. Use this when R is the region between two curves $y = f(x)$ and $y = g(x)$ for $a \leq x \leq b$, and it is rotated about a HORIZONTAL line. If $y = c$ is the axis of revolution, then $r_1 = f(x) - c$ and $r_2 = g(x) - c$.

   (c) **Cylindrical Shells (6.3).** $V = \int_a^b 2\pi r h \, dx$. Use this method when R is the region between two curves $y = f(x)$ and $y = g(x)$ for $a \leq x \leq b$, and it is rotated about a VERTICAL line. If $x = c$ is the axis of revolution, then $r = x - c$ and $h = f(x) - g(x)$.

   (d) **NOTE:** If the region is between two curves with equations $x = f(y)$ and $x = g(y)$, then you should integrate with respect to y. In this case, if the axis of revolution is Vertical, use Washers or Disks. If it is horizontal, use Cylindrical Shells.

3. **Work (6.4).** Calculate the work done moving an object with a variable force $F(x)$ from $x = a$ to $x = b$. You may also need to find an equation for $F(x)$ from a word problem. ((**6.4** # 15, 29))

4. **Average Value (6.5).** Calculate the average value of a function $f(x)$ on an interval $[a, b]$. (**7.1** # 59)

5. **Arclength (8.1).** Calculate (or at least set up an integral for) the arclength of a segment of a curve $y = f(x)$ (or $x = g(y)$). (**8.1** # 9, 11, 19)

6. **Surface Area (8.2).** Calculate (or at least set up integrals for) the surface area of the surface obtained by rotating a curve $y = f(x)$ about the x- or y-axis. (**8.2** # 3, 5, 11, 26)

7. **Hydrostatic Force (8.3).** Compute the hydrostatic force on the side of an object submerged in a liquid. (**8.3** # 11, 15)

Earlier Topics

8. **Integration Techniques (7.1-7.5).** Section 7.5 gives a good overview of integration strategies.

   (a) **Integration Formulas.** Review the table on p. 506. You need to know numbers 1-10, 11, and 13; while 12, 14, 17 and 18 are also useful.

   (b) **Substitution (5.5).** Remember the two guidelines for choosing u:

      1) u should correspond to the "inside" function in a composition; and
      2) $du = u'(x)\ dx$ should appear in the integrand, or at least be expressible in terms of u.

      DON'T FORGET to either change the limits of integration OR convert the antiderivative back into terms of x. (**5.5** 23, 43, 61, **7.5** # 5, 19)

   (c) **Integration by Parts (7.1).** Know when to use integration by parts, and how to choose u and dv (eg., u = L.I.A.T.E.). Be aware that some problems require two applications of integration by parts, or some combination of u-substitution and integration by parts. (**7.1** # 7, 23, 25, 29, 31, 35)

   (d) **Trigonometric Integrals (7.2).** Be able to integrate $\int \sin^m x \cos^n x\ dx$ and simple variations (like replacing the x's with $(3x)$ or when $m = 3$ and $n = 1/2$.) (**7.2** # 1, 5, 9, 15)

   (e) **Trigonometric Substitution (7.3).** Know when to make the substitution $x = a \sin\theta$ to simplify an integral, and how to convert the antiderivative back into terms of x by drawing a right triangle. (**7.3** # 7, 11, 29 (do u-sub. first))

   (f) **Integrating Rational Functions (7.4).** Know how to use long division of polynomials and the method of partial fractions to algebraically simplify rational functions, so that they can be integrated easily. (**7.4** # 8, 10, 13, 47)

9. **Improper Integrals (7.8).** Be able to recognize an improper integral. In particular, when you see any definite integral $\int_a^b f(x)\ dx$, you should check to see if $f(x)$ has a vertical asymptote between a and b. More importantly, you should be able to evaluate an improper integral USING LIMITS. (**7.8** # 9, 13, 15, 27, 33)

10. **Comparison Theorem (7.8).** Determine whether an improper integral converges or diverges by using the Comparison Theorem. (**7.8** # 49. 51, 53)

11. Given a graph of a function $f(x)$, you should be able to sketch a graph of an antiderivative of $f(x)$. If the antiderivative is defined by an integral, eg. $F(x) = \int_0^x f(t)\, dt$, then you should be able to compute values of $F(x)$ by interpreting the integral as a "net area". (**4.10** #47; **5.3** # 3.)

12. You should be able to write an area or a definite integral as a limit of Riemann sums, and interpret a limit of Riemann sums as a definite integral. (**5.1** # 19; **5.2** # 17, 19, 29)

13. You should know what the Fundamental Theorem of Calculus, Parts I and II, say and how to use them to (1) find the derivative of a function defined as an integral, and (2) evaluate a definite integral. (**5.3** # 11, 49, 55)