Name: Solution

Perm No.:

Section Time :

Math 3B - Midterm 1

April 26, 2007

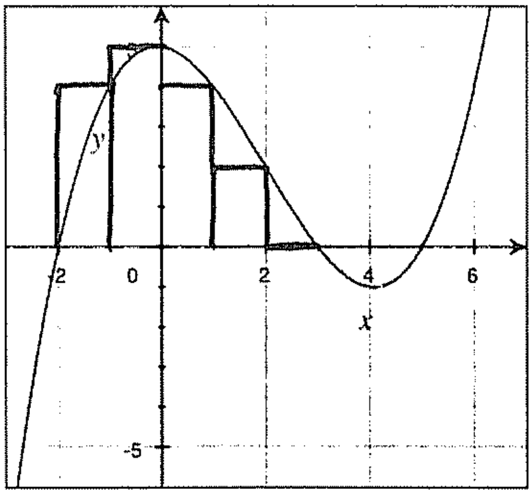
Instructions:

- This exam consists of 5 problems worth 6 points each, for a total of 30 possible points. There is also one extra credit question, worth 3 points.
- You must show all your work and fully justify your answers in order to recieve full credit. You may leave your answers in unsimplified form, unless the problem asks you to simplify. Partial credit will be awarded for work that is relevant and correct.
- No books, notes, calculators or other devices are allowed.
- Write your answers on the test itself, in the space alotted. You may attach additional pages if necessary.

1	
2	
3	
4	
5	
6	
Total	

1. The graph of a function $f(x)$ is given below.

   (a) Approximate the area under the graph of $f(x)$ from $x = -2$ to $x = 3$ using 5 rectangles and right endpoints. Sketch the rectangles on the graph.



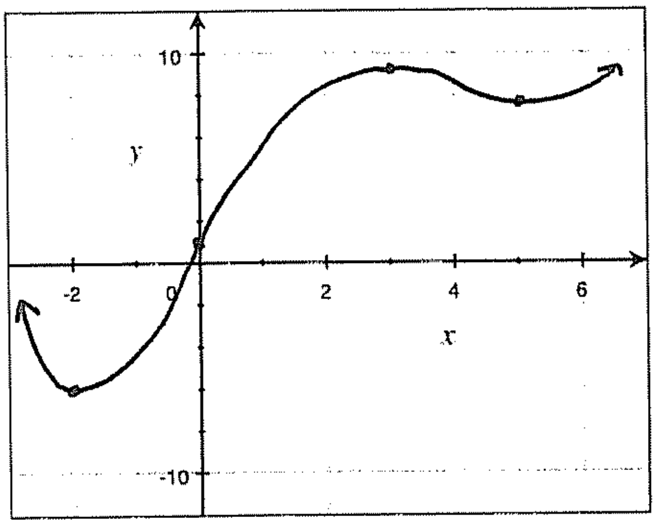
$\Delta x = \frac{3 - -2}{5} = 1$

Area $\approx \Delta x[f(-1) + f(0) + f(1) + f(2) + f(3)]$

$= 1[4 + 5 + 4 + 2 + 0]$

$= \boxed{15}$

   (b) Sketch a graph of the antiderivative $F(x)$ of $f(x)$ such that $F(0) = 1$.



$f(x)$ changes sign from $-$ to $+$ at $x = -2$, $x = 5$

$\Rightarrow F(x)$ has Rel. minima at $x = -2$, $x = 5$.

$f(x)$ changes sign from $+$ to $-$ at $x = 3$

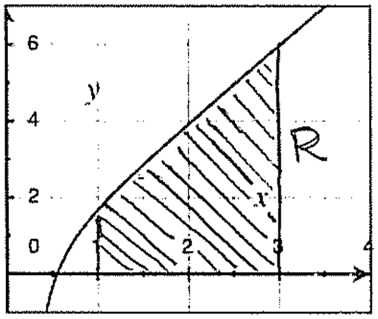
$\Rightarrow F(x)$ has a rel. max at $x = 3$

$F(x)$ increasing on $(-2, 3)$, $(6, \infty)$

$F(x)$ decreasing on $(-\infty, -2)$, $(3, 5)$

2. Let R be the region under the graph of $f(x) = 2x - \dfrac{1}{3x^2}$ (and above the x-axis) from $x = 1$ to $x = 3$.

(a) Express the area of R as a limit of Riemann sums.



$\Delta x = \frac{3-1}{n} = \frac{2}{n}$, $x_i = a + i\Delta x = 1 + i\left(\frac{2}{n}\right)$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\, f(x_i)$$

$$= \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left[2\left(1+\frac{2i}{n}\right) - \frac{1}{3\left(1+\frac{2i}{n}\right)^2}\right]}$$

(b) Calculate the area of R.

$$A = \int_1^3 \left(2x - \frac{1}{3x^2}\right) dx = \left(x^2 + \frac{1}{3}x^{-1}\right)\Big]_1^3$$

$$= \left(9 + \frac{1}{9}\right) - \left(1 + \frac{1}{3}\right)$$

$$= 8 + \frac{1}{9} - \frac{3}{9}$$

$$= \boxed{\frac{70}{9}}$$

3. Integrate.

(a) $\displaystyle\int \frac{x}{\sqrt{3+x^2}}\,dx$ $\displaystyle = \int \frac{du}{2\sqrt{u}} = \frac{1}{2}\int u^{-1/2}\,du$

let $u = 3 + x^2$

$du = 2x\,dx$

$$= \frac{\frac{1}{2}u^{1/2}}{1/2} + C$$

$$= \sqrt{u} + C$$

$$= \boxed{\sqrt{3+x^2} + C}$$

(b) $\displaystyle\int \frac{1+\sin x}{\cos^2 x}\,dx$ $\displaystyle = \int \frac{1}{\cos^2 x}\,dx + \int \frac{\sin x}{\cos^2 x}\,dx$

$$= \int \sec^2 x\,dx + \int \frac{\sin x}{\cos^2 x}\,dx$$

$$= \tan x + \int \frac{\sin x}{\cos^2 x}\,dx$$

let $u = \cos x$

$du = -\sin x\,dx$

$$= \int \frac{-du}{u^2} = -\frac{u^{-1}}{-1} + C$$

$$= \frac{1}{\cos x} + C$$

$$= \boxed{\tan x + \sec x + C}$$

4. Evaluate the definite integrals.

(a) $\int_0^5 |2-x|\,dx \;=\; \int_0^2 (2-x)\,dx + \int_2^5 (x-2)\,dx$

$|2-x| = \begin{cases} 2-x, & x \le 2 \\ x-2, & x > 2 \end{cases}$

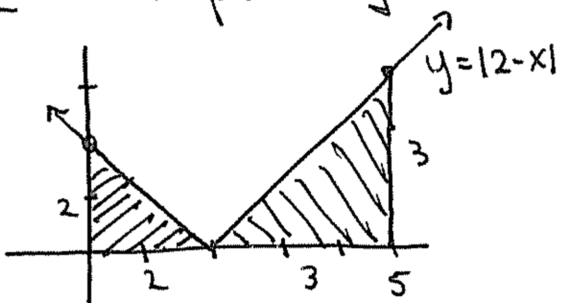
$= \left(2x - \frac{x^2}{2}\right)\Big]_0^2 + \left(\frac{x^2}{2} - 2x\right)\Big]_2^5$

$= (4-2) - 0 + \left(\frac{25}{2} - 10\right) - (2-4)$

$= 2 + \frac{5}{2} - (-2)$

$= \boxed{\frac{13}{2}}$

OR Graphically



Area $= \frac{1}{2}(2^2) + \frac{1}{2}(3^2) = \frac{13}{2}$.

(b) $\int_1^e \frac{1}{x + x(\ln x)^2}\,dx$

$= \int_1^e \frac{1}{x}\left(\frac{1}{1+(\ln x)^2}\right) dx \;=\; \int_{u=0}^1 \frac{1}{1+u^2}\,du$

let $u = \ln x$
$du = \frac{1}{x}dx$

$= \tan^{-1}(u)\Big]_0^1$

$= \tan^{-1}(1) - \tan^{-1}(0)$

$= \boxed{\frac{\pi}{4}}$

5. Suppose you drop a tennis ball from the top of a building and time that it takes 3 seconds to hit the ground. How tall is the building? (You may assume that the ball's acceleration due to gravity is $-32 ft/s^2$, and does not change during the fall.)

$a(t) = -32$

$v(t) = -32t + C$

$0 = v(0) = C$

$\Rightarrow v(t) = -32t.$

$S(t) = \int v(t)\,dt = -16t^2 + C$

$S(0) = C$ = initial height = height of building

$= ?$

$S(3) = 0 \Rightarrow -16(3)^2 + C = 0$

$\Rightarrow C = +16(3)^2$

$= +16 \cdot 9$

$= \boxed{144 \text{ ft.}}$

6. **Extra Credit.** If $f(x)$ is a continuous, even function (that is, $f(-x) = f(x)$ for all x), prove that $g(x) = \int_0^x f(t)\, dt$ is an odd function (that is, $g(-x) = -g(x)$ for all x).

$$g(-x) = \int_0^{-x} f(t)\, dt = \int_{u=0}^{x} f(-u)(-du)$$

let $u = -t$
$du = -dt$

$$= -\int_0^{x} f(u)\, du$$

$$= -\int_0^{x} f(t)\, dt$$

$$= -g(x).$$