FINAL EXAM

PLEASE PRINT

YOUR NAME:

YOUR TA's NAME:

TIME of DISCUSSION SECTION:

Books and calculators are not allowed. You may use one 3"x5" note card.

Problem 1. (10 points) A tank with a volume $48m^3$ is full with water. The water leaves the tank at a rate of $(2+4t)m^3/hr$. How much time will it take until the tank becomes half empty?

Problem 2. (10 points)
a) Evaluate $\int cos(\pi x + 2/\pi)dx$ b) Evaluate $\frac{d}{dx}(sin(2x)cos(3x+4))$.

Problem 3. (15 points)

a) Let $f(x)$ and $g(x)$ be two functions such that $f(5) = 1, f'(5) = 6, g(5) = -3, g'(5) = 2$. Find $(fg)'(5)$. (i.e. find the derivative of the function $f(x)g(x)$ at $x = 5$)

b) Suppose a rectangle initially has a width of 6cm and a height of 10cm. Suppose the width and height are both increasing at a rate of 2cm/sec. At what rate is the area increasing?

Problem 4. (10 points) Solve the differential equation $f''(x) = sin(x)$ subject to the initial conditions $f'(0) = 0$ and $f(0) = 1$.

Problem 5. (20 points) At noon, a rumor is started in a village. Let $y(t)$ be the fraction of the population that has heard the rumor by t hours after noon. Suppose the rate of spread of the rumor is proportional to the product of the fraction of the population that has heard the rumor and the fraction who have not heard the rumor.

a) Write a differential equation that is satisfied by y. [Note: At any given time, $y(t)$ is always between 0 and 1, so the fraction of the population who have not heard tghe rumor is given by $1 - y(t)$].

b) Find the general solution to this differential equation.

c) Suppose the village has 1000 people. At noon, 100 people have heard the rumor. At noon, the rumor is spreading at a rate of 270 people per hour. At what time will half of the village have heard the rumor? [Note: your final answer should involve a natural logarithm. You are not expected to give a numerical approximation of it.]

Problem 6. (10 points) Solve the differential equation $y' = 5y$, subject to the initial condition $y(0) = 2$.

Problem 7. (15 points)
a) Find the equation of the tangent plane to $z = x^3 - y^3$ at the point $(x, y) = (-2, 3)$.
b) Find the intersection point of this tangent plane with z-axis.
c) Apply the tangent approximation to find $(-2.001)^3 - (2.998)^3$.

Problem 8. (10 points)
Find the minimum of the function $f(x, y) = 2x^2 + y^2 - xy - 5x - y + 2$.