Math 34B Spring Quarter Midterm Examination
May 4, 2006

NAME: Answer Key (i).

TA & DISCUSSION SECTION: ....

You have 70 minutes in which to complete this examination. Attempt all of the questions.

Note that you will not be awarded full credit on a question unless your answer is clearly, carefully and neatly stated.

UP TO 5 BONUS POINTS (ADDED DIRECTLY TO YOUR SCORE ON THE EXAMINATION) WILL BE AWARDED FOR NEATLY AND CAREFULLY PRESENTED WORK!

Problem	Maximum Score	Score
1	9	
2	9	
3	9	
4	9	
5	6	
NEATNESS BONUS	5	
Total	42	

(1) Find the derivatives with respect to x of the following functions:

(i) $f(x) = 4\sin(2x) + 2x + 6$

$$f'(x) = 2\cdot 4\cos(2x) + 2$$
$$= 8\cos(2x) + 2$$

(ii) $f(x) = e^{2x}\sin(4x)$

$$f'(x) = \frac{d}{dx}(e^{2x})\sin(4x) + e^{2x}\cdot\frac{d}{dx}(\sin(4x))$$
$$= 2e^{2x}\sin(4x) + 4e^{2x}\cos(4x)$$
$$= 2e^{2x}(\sin(4x) + 2\cos(4x)).$$

(iii) $y = (f(x))^2$

$$y = f(x)\cdot f(x).$$
$$\therefore \frac{dy}{dx} = f'(x)\cdot f(x) + f(x)\cdot f'(x)$$
$$= 2f(x)f'(x).$$

(2) Find the following integrals:

(i) $\int 6e^{2t}\,dt$

$$\int 6e^{2t}\,dt = 3e^{2t} + C$$

(ii) $\int_{2}^{4} x^2\,dx$

$$\int_{2}^{4} x^2\,dx = \left[\frac{x^3}{3}\right]_2^4$$

$$= \frac{4^3}{3} - \frac{2^3}{3}$$

$$= \frac{56}{3}$$

(iii) $\int_{1}^{\pi} 2\,dx$

$$\int_{1}^{\pi} 2\,dx = \left[2x\right]_1^{\pi}$$

$$= 2\pi - 2.$$

(3) Consider the function $f(t) = 4\cos(2t+2)$, where t denotes time measured in seconds.

(a) What is the period of $f(t)$?

The period of $f(t)$ is π seconds.

(b) What is the frequency of $f(t)$?

The frequency of $f(t)$ is $\frac{1}{\pi}$ Herz.

(c) What is the amplitude of $f(t)$?

The amplitude of $f(t)$ is equal to 4.

(4) The height above the ground in metres of a rocket t seconds after being launched is $h(t) = 10t + 2t^2$.

(a) What is the velocity of the rocket after t seconds?

Let $v(t)$ = velocity of the rocket after t seconds.
Then

$$v(t) = \frac{dh}{dt} = \frac{d}{dt}(10t + 2t^2)$$
$$= 10 + 4t \text{ m/s.}$$

(b) How many seconds after the launch is the velocity of the rocket equal to $50\,m/s$?

The velocity of the rocket is 50 m/s when

$$10 + 4t = 50$$
$$\therefore\quad 4t = 50 - 10$$
$$= 40$$
$$\therefore\quad t = 10 \text{ seconds.}$$

Hence the velocity of the rocket is equal to 50 m/s after 10 seconds.

(c) What is the acceleration of the rocket t seconds after being launched?

Let $a(t)$ = acceleration of the rocket after t seconds.
Then

$$a(t) = \frac{dv}{dt}$$
$$= \frac{d}{dt}(10 + 4t)$$
$$= 4 \text{ m/s}^2.$$

(5) The radius of a circular oil slick is increasing at a rate of 1 metre per second. At time $t = 0$, the area of the slick is equal to $4\pi\, m^2$.

(a) Find the radius of the slick after t seconds.

Let $r(t)$ = radius of oil slick after t seconds.

Then $\frac{dr}{dt} = 1$, and so $r(t) = \int 1 \cdot dt = t + C$.

When $t=0$, the radius of the slick is $2m$ (since its area is $4\pi\, m^2$). So

$$2 = 0 + C,$$

and therefore $C = 0$.

$\therefore\ r(t) = t + 2$ metres.

(b) Find the rate at which the area of the slick is increasing after t seconds.

Let $A(t)$ = area of the slick after t seconds.

Then $A(t) = \pi r(t)^2$

$= \pi (t+2)^2$

$\therefore\ \frac{dA}{dt} = 2\pi(t+2)\ m^2/s.$

Hence the area of the slick is increasing at a rate of $2\pi(t+2)\ m^2/s$ after t seconds.